Fast Light, Fast Neutrinos?

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In certain media, light has been observed with group velocities faster than the speed of light. The recent OPERA report of superluminal 17 GeV neutrinos may describe a similar phenomenon.

Over the past decade, Boyd and others have observed light moving through certain media with group velocities faster than the speed of light in vacuum [1–3]. The OPERA collaboration [4] may have detected the neutrino analog of this "fast light" phenomenon.

The group velocity of an amplitude

$$A(x,t) = \int e^{i(k\cdot x - \omega t)}\, B(k)\, dk \tag{1}$$

is determined by the condition that the phase remain constant as a function of wave-number, $v_g = d\omega/dk$. When neutrinos traverse a medium with a complex index of refraction n, scattering makes the wave number k complex with a positive imaginary part. The real frequency ω is related to the complex wave number k by $k = n\,\omega/c$. If n_r is the real part of the index of refraction, then the group velocity is

$$v_g = \frac{d\omega}{dk_r} = \frac{c}{n_r + \omega\, dn_r/d\omega}. \tag{2}$$

The index of refraction n is related to the forward scattering amplitude f and the density N of scatterers by [5]

$$n(\omega) = 1 + \frac{2\pi c^2}{\omega^2} N f(\omega) \tag{3}$$

in which for simplicity I replaced k^2 by ω^2/c^2, which introduces an error of less than 10^{-19} for 17 GeV neutrinos with a mass of less than 2 eV/c^2 [4].

Group velocities faster than c can occur when the frequency ω is near a resonance in the total cross-section. For instance, if the amplitude for forward scattering is of the Breit-Wigner form

$$f(\omega) = f_0\, \frac{\Gamma/2}{\omega_0 - \omega - i\Gamma/2} \tag{4}$$

then the real part of the index of refraction is

$$n_r(\omega) = 1 + \frac{\pi c^2 N f_0 \Gamma(\omega_0 - \omega)}{\omega^2\left[(\omega-\omega_0)^2 + \Gamma^2/4\right]} \tag{5}$$

and by (2) the group velocity is

$$v_g = c\left[1 + \frac{\pi c^2 N f_0 \Gamma\,\omega_0}{\omega^2}\, \frac{\left[(\omega-\omega_0)^2 - \Gamma^2/4\right]}{\left[(\omega-\omega_0)^2 + \Gamma^2/4\right]^2}\right]^{-1} \tag{6}$$

which is superluminal if $(\omega-\omega_0)^2 < \Gamma^2/4$.

More generally, we may use the optical theorem and the regularized Kramers-Kronig formula

$$n_r(\omega) = 1 + \frac{cN}{\pi}\int_0^\infty \frac{\sigma_{\rm t}(\omega') - \sigma_{\rm t}(\omega)}{\omega'^2 - \omega^2}\, d\omega' \tag{7}$$

in which $\sigma_{\rm t}$ is the total cross-section to write the group velocity in terms of the principal part of an integral

$$\frac{c}{v_g(\omega)} = 1 + \frac{cN}{\pi} P\!\int_0^\infty \frac{\left[\sigma_{\rm t}(\omega') - \sigma_{\rm t}(\omega)\right](\omega'^2 + \omega^2)}{(\omega'^2 - \omega^2)^2}\, d\omega' \tag{8}$$

which shows the effect of scattering on group velocities. Just as the scattering of photons by atoms can cause fast [1, 3], slow [6], and even backward [7], light, so too the scattering of neutrinos by electrons and quarks may make neutrino group velocities that are faster or slower than the speed of light. The ν_μ-nucleon charged-current total cross-section rises linearly up to 300 GeV [8] and makes a positive contribution to the integral (8). Yet the OPERA Collaboration [4] may have discovered "fast neutrinos"—neutrinos with group velocities faster than the speed of light [9]. Their high group velocity $(v-c)/c = 2.48\times 10^{-5}$ may arise from a resonance in neutrino-electron and/or neutrino-quark scattering at an energy ω_0 somewhere near 17 GeV.

A group velocity faster than c doesn't violate special relativity, but a superluminal signal velocity would [2, 3].

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