Is Dark Energy a Cosmic Casimir Effect?

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Unknown short-distance effects cancel the quartic divergence of the zero-point energies. If this renormalization took effect in the early universe after the last phase transition and applied only to modes whose wavelengths λ were shorter than the Hubble length $H^{-1}(t^*)$ at that time, then the zero-point energies of the modes of longer wavelengths can approximately account for the present value of the dark-energy density. The model makes two predictions.

Observations of some 400 type-Ia supernovas suggest that the expansion of the universe is accelerating [1–3] as if subject to a negative pressure. Negative pressure is the derivative of the energy with respect to the volume at constant entropy

$$-p = \left.\frac{\partial U}{\partial V}\right|_S \tag{1}$$

and so the simplest explanation of this cosmic acceleration is that the energy density of empty space is positive. Data from WMAP and BAO tell us that this dark-energy density ρ_d is about 73 percent of the critical density $3H_0^2/8\pi G_N$ that makes the universe spatially flat [4] and so

$$\rho_d = 3.1 \times 10^{-47}\,\mathrm{GeV}^4. \tag{2}$$

Dark energy may be the energy of the ground state of whatever fundamental theory describes the universe. In quantum field theory to lowest order in the coupling constants, the ground-state energy is a sum over every type α of elementary field and all momenta k of the zero-point energies [5–9]

$$E_0 = \sum_{k,\alpha} (-1)^{2s_\alpha}\, g_\alpha\, \frac{1}{2}\,\hbar\omega_{k,\alpha} \tag{3}$$

in which s_α is the spin of the particle of field α, m_α its mass, $\omega_{k,\alpha} = c\sqrt{k^2 + c^2 m_\alpha^2}$ its energy, and the statistical weight g_α is $2s_\alpha + 1$ if $m_\alpha > 0$, 2 if $m_a = 0$ and $s_\alpha > 0$, and 1 if $m_a = s_\alpha = 0$.

The zero-point energy $E_{0\alpha}$ of every field is badly divergent. In natural units ($\hbar = c = 1$), it is

$$E_{0\alpha} = (-1)^{2s_\alpha}\, g_\alpha\, \frac{V}{2(2\pi)^3} \int \sqrt{k^2 + m_\alpha^2}\, d^3k. \tag{4}$$

Inserting a cutoff Λ, we find for a massless scalar field

$$E_{0s} = \frac{V}{16\pi^2}\,\Lambda^4. \tag{5}$$

If the cutoff Λ is the Planck mass $m_P = 1.22 \times 10^{19}$ GeV, then the contribution of a single massless scalar boson to the dark-energy density is

$$\rho_s = 1.4 \times 10^{74}\,\mathrm{GeV}^4. \tag{6}$$

The ratio of this estimate to the observed dark-energy density ρ_d is $\rho_s/\rho_d = 5 \times 10^{120}$, making ρ_s too big by more than 120 orders of magnitude. Clearly, zero-point energies must either cancel or be renormalized.

The dark-energy mechanism of this paper is based upon two related assumptions about zero-point energies. The first assumption is that at any time t, an expanding universe is sensitive only to the zero-point energies of wavelengths, let us say $\lambda = 1/k$, shorter than the Hubble distance $H^{-1}(t)$ at that time. The second assumption is that at some time t^* after the last phase transition in the early universe, unknown short-distance effects permanently renormalized and canceled the zero-point energy (4) of the modes to which the universe was then sensitive, that is, modes with wavelengths λ shorter than the Hubble distance $H^{-1}(t^*)$ at that time. These two assumptions imply that the dark-energy density at later times involves only those momenta that lie within the interval $H(t) < k < H(t^*)$

$$\rho_C(t) = \sum_{k,\alpha} \frac{(-1)^{2s_\alpha}\, g_\alpha}{16\pi^3} \int_{H(t)}^{H(t^*)} \sqrt{k^2 + m_\alpha^2}\, d^3k. \tag{7}$$

Because the limits on the momentum integration are both in the infrared, the energy $\sqrt{k^2 + m_\alpha^2}$ is dominated by the mass term except for neutrinos and massless particles. Thus, the dark-energy density is approximately

$$\begin{aligned} \rho_C(t) \;&\approx\; \sum_{k,\alpha} \frac{(-1)^{2s_\alpha}\, g_\alpha\, m_\alpha}{4\pi^2} \int_{H(t)}^{H(t^*)} k^2\, dk \\ &= \sum_{k,\alpha} \frac{(-1)^{2s_\alpha}\, g_\alpha\, m_\alpha}{12\pi^2} \left(H^3(t^*) - H^3(t)\right). \end{aligned} \tag{8}$$

If we knew the spectrum of masses and spins of the elementary particles and fields, then we could compute the Casimir energy density $\rho_C(t)$ as a function of the time t. Instead, I will define an effective excess $\langle M \rangle$ of boson over fermion masses as

$$\langle M \rangle \equiv \sum_{k,\alpha} (-1)^{2s_\alpha}\, g_\alpha\, m_\alpha. \tag{9}$$

The cosmic Casimir energy density at time t is then

$$\rho_C(t) \approx \frac{\langle M \rangle}{12\pi^2} \left(H^3(t^*) - H^3(t)\right). \tag{10}$$

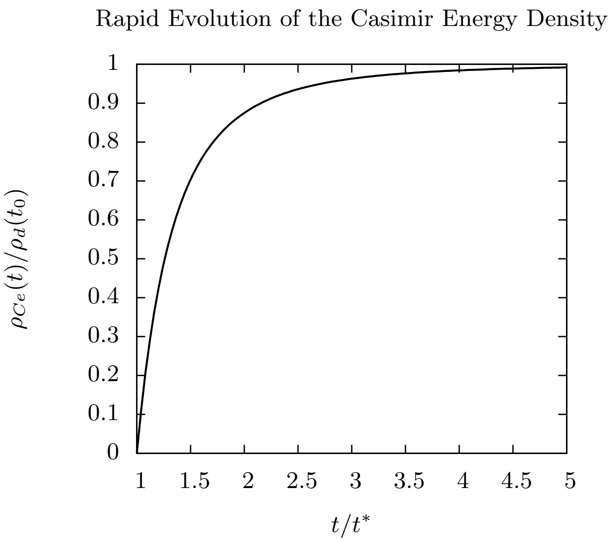

FIG. 1: The ratio of the energy density $\rho_C(t)$ of the cosmic Casimir effect (15) to the dark-energy density ρ_d is plotted for $\langle M\rangle\, c^2 = 1.03 \times 10^{14}$ GeV in the interval $1 < t/t^* < 5$.

The present value H_0 of the Hubble constant is so small that $\rho_C(t_0)$ is effectively

$$\rho_C(t_0) \approx \frac{\langle M\rangle\, H^3(t^*)}{12\pi^2}. \tag{11}$$

If the renormalization of the zero-point energies took place shortly after the QCD phase transition at $t^* = 10^{-5}$ s, then $1/H(t^*) = 2t^* = 2 \times 10^{-5}$ s, and so

$$\rho_C(t_0) \approx \frac{\langle M\rangle\, c^2\, \hbar^3}{96\pi^2\, t^{*3}} = 3.0 \times 10^{-61} \langle M\rangle\, c^2 \mathrm{GeV}^3. \tag{12}$$

Thus, if the effective excess bosonic mass (9) were

$$\langle M\rangle \approx 10^{14}\, \mathrm{GeV}/c^2 \tag{13}$$

then the present value of the Casimir energy density would approximately equal the dark-energy density

$$\rho_C(t_0) \approx \rho_d = 3.1 \times 10^{-47}\, \mathrm{GeV}^4. \tag{14}$$

This explanation of dark energy makes two predictions. The first is that the effective excess $\langle M\rangle$ of boson over fermion masses (9) is 10^{14} GeV/c^2 which is a plausible order of magnitude in a theory of grand unification.

The second prediction is that the dark-energy density varies with time as in (10), rapidly rising from zero at $t = t^* = 10^{-5}$ s to its present value ρ_d, as in the figure. It is therefore a kind of quintessence that does not increase the helium abundance [10]. More precisely, the very early universe after inflation is flat and dominated by radiation, and so the scale factor evolves as $a(t) \propto \sqrt{t}$, and the Hubble parameter as $H(t) = \dot{a}(t)/a(t) = 1/2t$. Thus, the Casimir energy density (10) rises as

$$\rho_C(t) \approx \frac{\langle M\rangle\, c^2 \hbar^3}{96\pi^2} \left(t^{*-3} - t^{-3} \right). \tag{15}$$

If the interval of integration is scaled to $a\, H(t) < k < a\, H(t^*)$ and the moment of renormalization to $b \times 10^{-5}$ s, then the predicted bosonic mass excess shifts to $b^3 \langle M\rangle / a^3$.

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