Exercises 513

generators $t^b$ of a representation of the gauge group. The generators obey the commutation relations

$$[t^a, t^b] = i f_{abc} t^c$$

in which the $f_{abc}$ are the structure constants of the gauge group. Show that under a gauge transformation (10.474)

$$A'_i = U A_i U^{-1} - (\partial_i U) U^{-1}$$

by the unitary matrix $U = \exp(-ig\lambda^a t^a)$ in which $\lambda^a$ is infinitesimal, the gauge-field matrix $A_i$ transforms as

$$-igA'_i t^a = -igA_i t^a - ig^2 f_{abc} \lambda^a A_i t^c + ig\partial_i \lambda^a t^a.$$  

(10.510)

Show further that the gauge field transforms as

$$A'^a_i = A^a_i - \partial_i \lambda^a - g f_{abc} A^b_i \lambda^c.$$  

(10.511)

10.43 Show that if the vectors $e_a(x)$ are orthonormal, then $e^{a\dagger} e_{c,i} = -e_{c,i}^{a\dagger} e_c$.

10.44 Use the identity of exercise 10.43 to derive the formula (10.489) for the nonabelian Faraday tensor.

10.45 Write Dirac’s action density in the explicitly hermitian form $L_D = -\frac{1}{4} \overline{\psi} \gamma^i \partial_i \psi - \frac{1}{2} \left[ \overline{\psi} \gamma^i \partial_i \psi \right]^\dagger$ in which the field $\psi$ has the invariant form $\psi = e_a \psi_a$ and $\overline{\psi} = i \psi^{i\dagger} \gamma^0$. Use the identity $\left[ \overline{\psi}_a \gamma^i \psi_b \right]^\dagger = - \overline{\psi}_b \gamma^i \psi_a$ to show that the gauge-field matrix $A_i$ defined as the coefficient of $e_a \gamma^i$ as in $\overline{\psi}_a \gamma^i (\partial_i + i A_{iab}) \psi_b$ is hermitian $A'^{a}_{iab} = A_i^{a}$. 
