extraordinary resource, as is Rubinstein's *Simulation and the Monte Carlo Method* (Rubinstein and Kroese, 2007).

Exercises

14.1 Go to Michael Creutz's website (latticeguy.net/lattice.html) and get his C-code for Z_2 lattice gauge theory. Compile and run it, and make a graph that exhibits strong hysteresis as you raise and lower $\beta = 1/kT$.

14.2 Modify his code and produce a graph showing the coexistence of two phases at the critical coupling $\beta_t = 0.5 \ln(1 + \sqrt{2})$. Hint: Do a cold start and then 100 updates at β_t, then do a random start and do 100 updates at β_t. Plot the values of the action against the update number 1, 2, 3, ... 100.

14.3 Modify Creutz's C code for Z_2 lattice gauge theory so as to be able to vary the dimension d of space-time. Show that for $d = 2$, there's no hysteresis loop (there's no phase transition). For $d = 3$, show that any hysteresis loop is minimal (there's a second-order phase transition).

14.4 What happens when $d = 5$?

14.5 Use example 14.3 to compute the ten-dimensional integral

$$\mathcal{I} = \int \exp\left[-\left(x^2 + (x^2)^2\right)\right] d^{10}x \tag{14.29}$$

over $\mathbb{R}^{10}$ where $x^2 = x_1^2 + \cdots + x_{10}^2$.