generators t^b of a representation of the gauge group. The generators obey the commutation relations

$$[t^a, t^b] = i f_{abc} t^c \tag{11.511}$$

in which the f_{abc} are the structure constants of the gauge group. Show that under a gauge transformation (11.477)

$$A'_i = U A_i U^{-1} - (\partial_i U)\, U^{-1} \tag{11.512}$$

by the unitary matrix $U = \exp(-ig\lambda^a t^a)$ in which λ^a is infinitesimal, the gauge-field matrix A_i transforms as

$$-igA'^a_i t^a = -igA^a_i t^a - ig^2 f_{abc} \lambda^a A^b_i t^c + ig\partial_i \lambda^a t^a. \tag{11.513}$$

Show further that the gauge field transforms as

$$A'^a_i = A^a_i - \partial_i \lambda^a - g f_{abc} A^b_i \lambda^c. \tag{11.514}$$

11.43 Show that if the vectors $e_a(x)$ are orthonormal, then $e^{a\dagger} \cdot e_{c,i} = -e^{a\dagger}_{,i} \cdot e_c$.

11.44 Use the identity of exercise 11.43 to derive the formula (11.492) for the nonabelian Faraday tensor.

11.45 Using the tricks of section 11.35, show that $\delta\sqrt{g} = -\frac{1}{2}\sqrt{g}\, g_{ik}\, \delta g^{ik}$. This relation and the definition (11.354) $R = g^{ik} R^n_{ink}$ imply that the first-order change in the Einstein-Hilbert action is (11.381) apart from an irrelevant surface term (Carroll, 2003, chap 4.3) due to $g^{ik}\sqrt{g}\; \delta R^n_{ink}$.

11.46 Write Dirac's action density in the explicitly hermitian form $L_D = -\frac{1}{2}\overline{\psi}\gamma^i \partial_i \psi - \frac{1}{2}\left[\overline{\psi}\gamma^i \partial_i \psi\right]^\dagger$ in which the field ψ has the invariant form $\psi = e_a \psi_a$ and $\overline{\psi} = i\psi^\dagger \gamma^0$. Use the identity $\left[\overline{\psi}_a \gamma^i \psi_b\right]^\dagger = -\overline{\psi}_b \gamma^i \psi_a$ to show that the gauge-field matrix A_i defined as the coefficient of $\overline{\psi}_a \gamma^i \psi_b$ as in $\overline{\psi}_a \gamma^i (\partial_i + iA_{iab})\psi_b$ is hermitian $A^*_{iab} = A_{iba}$.