The eight states of the system $|t,u,v\rangle \equiv (a_1^\dagger)^t(a_2^\dagger)^u(a_3^\dagger)^v|0,0,0\rangle$. We can represent them by eight 8-vectors each of which has seven 0's with a 1 in position $4t+2u+v+1$. How big should the matrices that represent the creation and annihilation operators be? Write down the three matrices that represent the three creation operators.

1.38 Show that the Schwarz inner product (1.430) is degenerate because it can violate (1.79) for certain density operators and certain pairs of states.

1.39 Show that the Schwarz inner product (1.431) is degenerate because it can violate (1.79) for certain density operators and certain pairs of operators.

1.40 The coherent state $|\{\alpha_k\}\rangle$ is an eigenstate of the annihilation operator a_k with eigenvalue α_k for each mode k of the electromagnetic field, $a_k|\{\alpha_k\}\rangle = \alpha_k|\{\alpha_k\}\rangle$. The positive-frequency part $E_i^{(+)}(x)$ of the electric field is a linear combination of the annihilation operators

$$E_i^{(+)}(x) = \sum_k a_k \, \mathcal{E}_i^{(+)}(k) \, e^{i(kx-\omega t)}. \tag{1.453}$$

Show that $|\{\alpha_k\}\rangle$ is an eigenstate of $E_i^{(+)}(x)$ as in (1.442) and find its eigenvalue $\mathcal{E}_i(x)$.

1.41 Show that if X is a non defective, nonsingular square matrix, then the variation of the logarithm of its determinant is $\delta \ln(\det X) = \mathrm{Tr}(X^{-1}\delta X)$.