The full action of a stretched field is
\[ S(\phi_L) = \int d^d x \left( \frac{1}{2} (\partial \phi)^2 + \sum_n g_{d,n}(L) \phi^n \right) \] (17.48)
in which
\[ g_{d,n}(L) = L^d A^n(L) g_n = L^{d-n(d-2)/2} g_{d,n}. \] (17.49)

The beta-function
\[ \beta(g_{d,n}) = \frac{L}{g_{d,n}(L)} \frac{dg_{d,n}(L)}{dL} = d - n(d - 2)/2 \] (17.50)
is just the exponent of the coupling “constant” \( g_{d,n}(L) \). If it is positive, then the coupling constant \( g_{d,n}(L) \) gets stronger as \( L \to \infty \); such couplings are called relevant. Couplings with vanishing exponents are insensitive to changes in \( L \) and are marginal. Those with negative exponents shrink with increasing \( L \); they are irrelevant.

The coupling constant \( g_{d,n,p} \) of a term with \( p \) derivatives and \( n \) powers of the field \( \phi \) in a space of \( d \) dimensions varies as
\[ g_{d,n,p}(L) = L^d A^n(L) L^{-p} g_{n,p} = L^{d-n(d-2)/2-p} g_{d,n,p}. \] (17.51)

**Example 17.3 (QCD)** In quantum chromodynamics, there is a cubic term \( g f_{abc} A^a_0 A^b_i \partial_0 A^c_i \) which in effect looks like \( g f_{abc} \phi_0 \phi_b \phi_c \). Is it relevant? Well, if we stretch space but not time, then the time derivative has no effect, and \( d = 3 \). So the cubic, \( n = 3 \), grows as \( L^{3/2} \)
\[ g_{3,3,0}(L) = L^{d-n(d-2)/2} g_{3,3,0} = L^{3/2} g_{3,3,0}. \] (17.52)
Since this cubic term drives asymptotic freedom, its strengthening as space is stretched by the dimensionless factor \( L \) may point to a qualitative explanation of confinement. For if \( g_{3,3,0}(L) \) grows with distance as \( L^{3/2} \), then \( \alpha_s(L) = g_{3,3,0}^2(L)/4\pi \) grows as \( L^3 \), and so the strength \( \alpha_s(Lr)/(Lr)^2 \) of the force between two quarks separated by a distance \( Lr \) grows linearly with \( L \)
\[ F(Lr) = \frac{\alpha_s(Lr)}{(Lr)^2} = \frac{L^3 \alpha_s(r)}{(Lr)^2} = L \frac{\alpha_s(r)}{r^2} \] (17.53)
which may be enough for quark confinement.

On the other hand, if we stretch both space and time, then the cubic \( g_{4,3,1}(L) \) and quartic \( g_{4,4,0}(L) \) couplings are marginal.