which contributes the terms $-\partial_\mu \omega_a^* \partial^\mu \omega_a$ and

$$-\partial_\mu \omega_a^* \, g \, f_{abc} \, A_b^\mu \omega_c = \partial_\mu \omega_a^* \, g \, f_{abc} \, A_c^\mu \omega_b \tag{16.254}$$

to the action density.

Thus we can do perturbation theory by using the modified action density

$$\mathcal{L}' = -\tfrac{1}{4} F_{a\mu\nu} F_a^{\mu\nu} - \tfrac{1}{2} \left(\partial_\mu A_a^\mu\right)^2 - \partial_\mu \omega_a^* \partial^\mu \omega_a + \partial_\mu \omega_a^* \, g \, f_{abc} \, A_c^\mu \omega_b - \overline{\psi} \left(\not{D} + m\right) \psi \tag{16.255}$$

in which $\not{D} \equiv \gamma^\mu D_\mu = \gamma^\mu (\partial_\mu - i g t^a A_{a\mu})$. The **ghost** field ω is a mathematical device, not a physical field describing real particles, which would be spinless fermions violating the spin-statistics theorem (example 10.19).

Further Reading

Quantum Field Theory (Srednicki, 2007), *The Quantum Theory of Fields I, II, & III* (Weinberg, 1995, 1996, 2005), and *Quantum Field Theory in a Nutshell* (Zee, 2010) all provide excellent treatments of path integrals.

Exercises

16.1 Derive the multiple gaussian integral (16.8) from (5.167).

16.2 Derive the multiple gaussian integral (16.12) from (5.166).

16.3 Show that the vector $\overline{Y}$ that makes the argument of the multiple gaussian integral (16.12) stationary is given by (16.13), and that the multiple gaussian integral (16.12) is equal to its exponential evaluated at its stationary point $\overline{Y}$ apart from a prefactor involving $\det iS$.

16.4 Repeat the previous exercise for the multiple gaussian integral (16.11).

16.5 Compute the double integral (16.23) for the case $V(q_j) = 0$.

16.6 Insert into the LHS of (16.54) a complete set of momentum dyadics $|p\rangle\langle p|$, use the inner product $\langle q|p\rangle = \exp(iqp\hbar)/\sqrt{2\pi\hbar}$, do the resulting Fourier transform, and so verify the free-particle path integral (16.54).

16.7 By taking the nonrelativistic limit of the formula (11.311) for the action of a relativistic particle of mass m and charge q, derive the expression (16.55) for the action a nonrelativistic particle in an electromagnetic field with no scalar potential.

16.8 Show that for the hamiltonian (16.60) of the simple harmonic oscillator the action $S[q_c]$ of the classical path is (16.67).

16.9 Show that the harmonic-oscillator action of the loop (16.68) is (16.69).

16.10 Show that the harmonic-oscillator amplitude (16.72) for $q' = 0$ and $q'' = q$ reduces as $t \to 0$ to the one-dimensional version of the free-particle amplitude (16.54).

16.11 Work out the path-integral formula for the amplitude for a mass m initially at rest to fall to the ground from height h in a gravitational field of local acceleration g to lowest order and then including loops up to an overall constant. Hint: use the technique of section 16.7.

16.12 Show that the action (16.74) of the stationary solution (16.77) is (16.79).

16.13 Derive formula (16.132) for the action $S_0[\phi]$ from (16.130 & 16.131).

16.14 Derive identity (16.136). Split the time integral at $t = 0$ into two halves, use

$$\epsilon \, e^{\pm \epsilon t} = \pm \frac{d}{dt} \, e^{\pm \epsilon t} \tag{16.256}$$

and then integrate each half by parts.

16.15 Derive the third term in equation (16.138) from the second term.

16.16 Use (16.143) and the Fourier transform (16.144) of the external current j to derive the formula (16.145) for the modified action $S_0[\phi, \epsilon, j]$.

16.17 Derive equation (16.147) from equations (16.145) and (16.146).

16.18 Derive the formula (16.148) for $Z_0[j]$ from the expression (16.147) for $S_0[\phi, \epsilon, j]$.

16.19 Derive equations (16.149 & 16.150) from formula (16.148).

16.20 Derive equation (16.154) from the formula (16.149) for $Z_0[j]$.

16.21 Show that the time integral of the Coulomb term (16.159) is the term that is quadratic in j^0 in the number F defined by (16.164).

16.22 By following steps analogous to those the led to (16.150), derive the formula (16.177) for the photon propagator in Feynman's gauge.

16.23 Derive expression (16.192) for the inner product $\langle \zeta | \theta \rangle$.

16.24 Derive the representation (16.195) of the identity operator I for a single fermionic degree of freedom from the rules (16.182 & 16.185) for Grassmann integration and the anticommutation relations (16.178 & 16.184).

16.25 Derive the eigenvalue equation (16.200) from the definition (16.198 & 16.199) of the eigenstate $|\theta\rangle$ and the anticommutation relations (16.196 & 16.197).

16.26 Derive the eigenvalue relation (16.213) for the Fermi field $\psi_m(\boldsymbol{x}, t)$ from the anticommutation relations (16.209 & 16.210) and the definitions (16.211 & 16.212).

16.27 Derive the formula (16.214) for the inner product $\langle \chi' | \chi \rangle$ from the definition (16.212) of the ket $|\chi\rangle$.