We now make the gauge transformation
\[ A'_b(x) = A_b(x) + \partial_b \Lambda(x) \quad \text{and} \quad \psi'(x) = e^{i q \Lambda(x)} \psi(x) \quad (16.169) \]
and replace the fields \( A_b(x) \) and \( \psi(x) \) everywhere in the numerator and (separately) in the denominator in the ratio (16.168) of path integrals by their gauge transforms (16.169) \( A'_b(x) \) and \( \psi'(x) \). This change of variables changes nothing; it’s like replacing \( \mathcal{R} \int f(x) \, dx \) by \( \mathcal{R} \int f(y) \, dy \), and so
\[
\langle \Omega | \mathcal{T} [\mathcal{O}_1 \ldots \mathcal{O}_n] | \Omega \rangle = \langle \Omega | \mathcal{T} [\mathcal{O}_1 \ldots \mathcal{O}_n] | \Omega' \rangle \quad (16.170)
\]
in which the prime refers to the gauge transformation (16.169).

We’ve seen that the action \( S \) is gauge invariant. So is the measure \( DA \, D\psi \), and we now restrict ourselves to operators \( \mathcal{O}_1 \ldots \mathcal{O}_n \) that are gauge invariant. So in the right-hand side of equation (16.170), the replacement of the fields by their gauge transforms affects only the term \( \delta [\nabla \cdot A] \) that enforces the Coulomb-gauge condition
\[
\langle \Omega | \mathcal{T} [\mathcal{O}_1 \ldots \mathcal{O}_n] | \Omega \rangle = \frac{\int \mathcal{O}_1 \ldots \mathcal{O}_n \, e^{i S} \, \delta [\nabla \cdot A + \Delta \Lambda] \, DA \, D\psi}{\int e^{i S} \, \delta [\nabla \cdot A + \Delta \Lambda] \, DA \, D\psi}. \quad (16.171)
\]

We now have two choices. If we integrate over all gauge functions \( \Lambda(x) \) in both the numerator and the denominator of this ratio (16.171), then apart from over-all constants that cancel, the mean value in the vacuum of the time-ordered product is the ratio
\[
\langle \Omega | \mathcal{T} [\mathcal{O}_1 \ldots \mathcal{O}_n] | \Omega \rangle = \frac{\int \mathcal{O}_1 \ldots \mathcal{O}_n \, e^{i S} \, DA \, D\psi}{\int e^{i S} \, DA \, D\psi} \quad (16.172)
\]
in which we integrate over all matter fields, gauge fields, and gauges. That is, we do not fix the gauge.

The analogous formula for the euclidean time-ordered product is
\[
\langle \Omega | \mathcal{T}_e [\mathcal{O}_1 \ldots \mathcal{O}_n] | \Omega \rangle = \frac{\int \mathcal{O}_1 \ldots \mathcal{O}_n \, e^{- S_e} \, DA \, D\psi}{\int e^{- S_e} \, DA \, D\psi} \quad (16.173)
\]
in which the euclidean action \( S_e \) is the space-time integral of the energy density. This formula is quite general; it holds in nonabelian gauge theories and is important in lattice gauge theory.

Our second choice is to multiply the numerator and the denominator