The **time-ordered product** of two fields, as in (16.87), is the sum

$$\mathcal{T}\left[\phi(x_1)\phi(x_2)\right] = \theta(x_1^0 - x_2^0)\phi(x_1)\phi(x_2) + \theta(x_2^0 - x_1^0)\phi(x_2)\phi(x_1). \quad (16.120)$$

Between two factors of $\exp(-itH)$, it is for $t_1 > t_2$

$$e^{-itH}\mathcal{T}\left[\phi(x_1)\phi(x_2)\right]e^{-itH} = e^{-i(t-t_1)H}\phi(\boldsymbol{x}_1,0)e^{-i(t_1-t_2)H}\phi(\boldsymbol{x}_2,0)e^{-i(t+t_2)H}.$$

So by the logic that led to the path-integral formulas (16.112) and (16.117), we can write a matrix element of the time-ordered product (16.120) as

$$\langle\phi''|e^{-itH}\mathcal{T}\left[\phi(x_1)\phi(x_2)\right]e^{-itH}|\phi'\rangle = \int_{\phi'}^{\phi''}\phi(x_1)\,\phi(x_2)\,e^{iS[\phi]}\,D\phi \quad (16.121)$$

in which we integrate over fields that go from ϕ' at time $-t$ to ϕ'' at time t. The time-ordered product of any combination of fields is then

$$\langle\phi''|e^{-itH}\mathcal{T}\left[\phi(x_1)\dots\phi(x_n)\right]e^{-itH}|\phi'\rangle = \int\phi(x_1)\dots\phi(x_n)\,e^{iS[\phi]}\,D\phi. \quad (16.122)$$

Like the position eigenstates $|q'\rangle$ of quantum mechanics, the eigenstates $|\phi'\rangle$ are states of infinite energy that overlap most states. Yet we often are interested in the ground state $|0\rangle$ or in states of a few particles. To form such matrix elements, we multiply both sides of equations (16.117 & 16.122) by $\langle 0|\phi''\rangle\langle\phi'|0\rangle$ and integrate over ϕ' and ϕ''. Since the ground state is a normalized eigenstate of the hamiltonian $H|0\rangle = E_0|0\rangle$ with eigenvalue E_0, we find from (16.117)

$$\begin{aligned}\int\langle 0|\phi''\rangle\langle\phi''|e^{-i2tH}|\phi'\rangle\langle\phi'|0\rangle D\phi'' D\phi' &= \langle 0|e^{-i2tH}|0\rangle \quad (16.123)\\ &= e^{-i2tE_0} = \int\langle 0|\phi''\rangle e^{iS[\phi]}\langle\phi'|0\rangle\,D\phi D\phi'' D\phi'\end{aligned}$$

and from (16.122) suppressing the differentials $D\phi'' D\phi'$

$$e^{-2itE_0}\langle 0|\mathcal{T}\left[\phi(x_1)\dots\phi(x_n)\right]|0\rangle = \int\langle 0|\phi''\rangle\phi(x_1)\dots\phi(x_n)\,e^{iS[\phi]}\langle\phi'|0\rangle\,D\phi. \quad (16.124)$$

The mean value in the ground state of a time-ordered product of field operators is then a ratio of these path integrals

$$\langle 0|\mathcal{T}\left[\phi(x_1)\dots\phi(x_n)\right]|0\rangle = \frac{\int\langle 0|\phi''\rangle\,\phi(x_1)\dots\phi(x_n)\,e^{iS[\phi]}\langle\phi'|0\rangle\,D\phi}{\int\langle 0|\phi''\rangle\,e^{iS[\phi]}\langle\phi'|0\rangle\,D\phi} \quad (16.125)$$

in which factors involving E_0 have canceled and the integration is over all