and is positive for all functions $h(t)$. The stationary classical trajectory

$$q(t) = \frac{t - t_1}{t_2 - t_1} q(t_2) + \frac{t_2 - t}{t_2 - t_1} q(t_1)$$  \hspace{1cm} (15.24)$$
is a minimum of the action $S_0[q]$.

The second functional derivative of the action $S[q]$ (15.2) is

$$\delta^2 S[q][h] = \frac{d^2}{d\epsilon^2} \left. \int_{t_1}^{t_2} dt \left[ \frac{m}{2} \left( \frac{dq(t)}{dt} + \epsilon \frac{dh(t)}{dt} \right)^2 - V(q(t) + \epsilon h(t)) \right] \right|_{\epsilon=0}$$

$$= \int_{t_1}^{t_2} dt \left[ m \left( \frac{dh(t)}{dt} \right)^2 - \frac{\partial^2 V(q(t))}{\partial q^2(t)} h^2(t) \right]$$  \hspace{1cm} (15.25)$$

and it can be positive, zero, or negative. Chaos sometimes arises in systems of several particles when the second variation of $S[q]$ about a stationary path is negative, $\delta^2 S[q][h] < 0$ while $\delta S[q][h] = 0$.

The $n$th functional derivative is defined as

$$\delta^n G[f][h] = \frac{d^n}{d\epsilon^n} G[f + \epsilon h] \bigg|_{\epsilon=0}. \hspace{1cm} (15.26)$$

The $n$th functional derivative of the functional (15.21) is

$$\delta^n G_N[f][h] = \frac{N!}{(N-n)!} \int f^{N-n}(x) h^n(x) dx. \hspace{1cm} (15.27)$$

15.4 Functional Taylor Series

It follows from the Taylor-series theorem (section 4.6) that

$$e^{\delta G[f][h]} = \sum_{n=0}^{\infty} \frac{\delta^n}{n!} G[f][h] = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n}{d\epsilon^n} G[f + \epsilon h] \bigg|_{\epsilon=0} = G[f + h] \hspace{1cm} (15.28)$$

which illustrates an advantage of the present mathematical notation.

The functional $S_0[q]$ of Eq.(15.1) provides a simple example of the func-