evolution more ergodic, which is why most complex modern organisms use sexual reproduction.

Other genomic changes occur when a virus inserts its DNA into that of a cell and when transposable elements (transposons) of DNA move to different sites in a genome.

In evolution, the rest of the Metropolis step is done by the new individual: if he or she survives and multiplies, then the change is accepted; if he or she dies without progeny, then the change is rejected. Evolution is slow, but it has succeeded in turning a soup of simple molecules into humans with brains of 100 billion neurons, each with 1000 connections to other neurons.

John Holland and others have incorporated analogs of these Metropolis steps into Monte Carlo techniques called **genetic algorithms** for solving wide classes of problems (Holland, 1975; Vose, 1999; Schmitt, 2001).

Evolution also occurs at the cellular level when a cell mutates enough to escape the control imposed on its proliferation by its neighbors and transforms into a cancer cell.

Further Reading

The classic *Quarks, Gluons, and Lattices* (Creutz, 1983) is a marvelous introduction to the subject; his website (latticeguy.net/lattice.html) is an extraordinary resource, as is Rubinstein's *Simulation and the Monte Carlo Method* (Rubinstein and Kroese, 2007).

Exercises

14.1 Go to Michael Creutz's website (latticeguy.net/lattice.html) and get his C-code for Z_2 lattice gauge theory. Compile and run it, and make a graph that exhibits strong hysteresis as you raise and lower $\beta = 1/kT$.

14.2 Modify his code and produce a graph showing the coexistence of two phases at the critical coupling $\beta_t = 0.5 \ln(1 + \sqrt{2})$. Hint: Do a cold start and then 100 updates at β_t, then do a random start and do 100 updates at β_t. Plot the values of the action against the update number 1, 2, 3, $\ldots$ 100.

14.3 Modify Creutz's C code for Z_2 lattice gauge theory so as to be able to vary the dimension d of space-time. Show that for $d = 2$, there's no hysteresis loop (there's no phase transition). For $d = 3$, show that any hysteresis loop is minimal (there's a second-order phase transition).

14.4 What happens when $d = 5$?