identically distributed random variables of zero mean and variance σ^2 gives rise to Pearson's **chi-squared distribution** on $(0, \infty)$

$$P_{n,P}(x,\sigma)dx = \frac{\sqrt{2}}{\sigma}\frac{1}{\Gamma(n/2)}\left(\frac{x}{\sigma\sqrt{2}}\right)^{n-1} e^{-x^2/(2\sigma^2)}dx \qquad (13.200)$$

which for $x = v$, $n = 3$, and $\sigma^2 = kT/m$ is (exercise 13.29) the Maxwell-Boltzmann distribution (13.100). In terms of $\chi = x/\sigma$, it is

$$P_{n,P}(\chi^2/2)\, d\chi^2 = \frac{1}{\Gamma(n/2)}\left(\frac{\chi^2}{2}\right)^{n/2-1} e^{-\chi^2/2} d\left(\chi^2/2\right). \qquad (13.201)$$

It has mean and variance

$$\mu = n \quad \text{and} \quad \sigma^2 = 2n \qquad (13.202)$$

and is used in the chi-squared test (Pearson, 1900).

Personal income, the amplitudes of catastrophes, the price changes of financial assets, and many other phenomena occur on both small and large scales. **Lévy** distributions describe such multi-scale phenomena. The characteristic function for a symmetric Lévy distribution is for $\nu \le 2$

$$\tilde{L}_\nu(k, a_\nu) = \exp\left(-a_\nu |k|^\nu\right). \qquad (13.203)$$

Its inverse Fourier transform (13.174) is for $\nu = 1$ (exercise 13.30) the **Cauchy** or **Lorentz** distribution

$$L_1(x, a_1) = \frac{a_1}{\pi(x^2 + a_1^2)} \qquad (13.204)$$

and for $\nu = 2$ the gaussian

$$L_2(x, a_2) = P_G(x, 0, \sqrt{2a_2}) = \frac{1}{2\sqrt{\pi a_2}} \exp\left(-\frac{x^2}{4a_2}\right) \qquad (13.205)$$

but for other values of ν no simple expression for $L_\nu(x, a_\nu)$ is available. For $0 < \nu < 2$ and as $x \to \pm\infty$, it falls off as $|x|^{-(1+\nu)}$, and for $\nu > 2$ it assumes negative values, ceasing to be a probability distribution (Bouchaud and Potters, 2003, pp. 10–13).

13.14 The Central Limit Theorem and Jarl Lindeberg

We have seen in sections (13.7 & 13.8) that unbiased fluctuations tend to distribute the position and velocity of molecules according to Gauss's distribution (13.75). Gaussian distributions occur very frequently. The **central limit theorem** suggests why they occur so often.