gives $P(A|B,C) = P(A \cap B \cap C)/P(B \cap C)$. If we multiply (13.3) by $P(B)$, we get

$$P(A,B) = P(A \cap B) = P(B|A)\,P(A) = P(A|B)\,P(B). \tag{13.4}$$

Combination of (13.3 & 13.4) gives **Bayes's theorem** (Riley et al., 2006, p. 1132)

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)} \tag{13.5}$$

(Thomas Bayes, 1702–1761).

If the set B of outcomes or events is contained in the union of N mutually exclusive sets A_j of outcomes, then we must sum over them

$$P(B) = \sum_{j=1}^{N} P(B|A_j)\,P(A_j). \tag{13.6}$$

The probabilities $P(A_j)$ are called ***a priori*** probabilities. In this case, Bayes's theorem is (Roe, 2001, p. 119)

$$P(A_k|B) = \frac{P(B|A_k)\,P(A_k)}{\sum_{j=1}^{N} P(B|A_j)\,P(A_j)}. \tag{13.7}$$

If there are several B's, then a third form of Bayes's theorem is

$$P(A_k|B_\ell) = \frac{P(B_\ell|A_k)\,P(A_k)}{\sum_{j=1}^{N} P(B_\ell|A_j)\,P(A_j)}. \tag{13.8}$$

Example 13.1 (The Low-Base-Rate Problem) Suppose the incidence of a rare disease in a population is $P(D) = 0.001$. Suppose a test for the disease has a **sensitivity** of 99%, that is, the probability that a carrier will test positive is $P(+|D) = 0.99$. Suppose the test also is highly **selective** with a false-positive rate of only $P(+|N) = 0.005$. Then the probability that a random person in the population would test positive is by (13.6)

$$P(+) = P(+|D)\,P(D) + P(+|N)\,P(N) = 0.005993. \tag{13.9}$$

And by Bayes's theorem (13.5), the probability that a person who tests positive actually has the disease is only

$$P(D|+) = \frac{P(+|D)\,P(D)}{P(+)} = \frac{0.99 \times 0.001}{0.005993} = 0.165 \tag{13.10}$$

and the probability that a person testing positive actually is healthy is $P(N|+) = 1 - P(D|+) = 0.835$.

Even with an excellent test, screening for rare diseases is problematic.