with $\phi_0^\dagger \phi_0 = m^2/\lambda$ so as to minimize their potential energy density $V(\phi)$. Their kinetic action $(D^i\phi)^\dagger D_i\phi = (\partial^i\phi + A^i\phi)^\dagger(\partial_i\phi + A_i\phi)$ then is in effect $\phi_0^\dagger \, A^i \, A_i\phi_0$. The gauge-field matrix $A^i_{ab} = i\, t^\alpha_{ab} A^i_\alpha$ is a linear combination of the generators t^α of the gauge group. So the action of the scalar fields contains the term $\phi_0^\dagger \, A^i \, A_i \, \phi_0 = -M^2_{\alpha\beta} \, A^i_\alpha \, A_{i\beta}$ in which the mass-squared matrix for the gauge fields is $M^2_{\alpha\beta} = \phi_0^{*a} \, t^\alpha_{ab} \, t^\beta_{bc} \, \phi^c_0$. This **Higgs mechanism** gives masses to those linear combinations $b_{\beta i} \, A_\beta$ of the gauge fields for which $M^2_{\alpha\beta} \, b_{\beta i} = m_i^2 b_{\alpha i} \neq 0$.

The Higgs mechanism also gives masses to the fermions. The mass term m in the Yang-Mills-Dirac action is replaced by something like $c\,\phi$ in which c is a constant, different for each fermion. In the vacuum and at low temperatures, each fermion acquires as its mass $c\,\phi_0$. On 4 July 2012, physicists at CERN's Large Hadron Collider announced the discovery of a Higgs-like particle with a mass near 126 GeV/c^2 (Peter Higgs 1929–).

11.51 Gauge Theory and Vectors

This section is optional on a first reading.

We can formulate Yang-Mills theory in terms of vectors as we did relativity. To accomodate noncompact groups, we will generalize the unitary matrices $U(x)$ of the Yang-Mills gauge group to nonsingular matrices $V(x)$ that act on n matter fields $\psi^a(x)$ as

$$\psi'^a(x) = \sum_{a=1}^n V^a{}_b(x)\,\psi^b(x). \tag{11.480}$$

The field

$$\Psi(x) = \sum_{a=1}^n e_a(x)\,\psi^a(x) \tag{11.481}$$

will be gauge invariant $\Psi'(x) = \Psi(x)$ if the vectors $e_a(x)$ transform as

$$e'_a(x) = \sum_{b=1}^n e_b(x)\,V^{-1b}{}_a(x). \tag{11.482}$$

In what follows, we will sum over repeated indices from 1 to n and often will suppress explicit mention of the space-time coordinates. In this compressed notation, the field Ψ is gauge invariant because

$$\Psi' = e'_a\,\psi'^a = e_b\,V^{-1b}{}_a\,V^a{}_c\,\psi^c = e_b\,\delta^b{}_c\,\psi^c = e_b\,\psi^b = \Psi \tag{11.483}$$

which is $e'^{\mathsf{T}}\psi' = e^{\mathsf{T}}V^{-1}V\psi = e^{\mathsf{T}}\psi$ in matrix notation.