Since the vectors $e_i$ are orthogonal, the metric is diagonal
\begin{equation}
    g_{ij} = e_i \cdot e_j = h_i^2 \delta_{ij}. \tag{11.154}
\end{equation}

The inverse metric
\begin{equation}
    g^{ij} = h_i^{-2} \delta_{ij} \tag{11.155}
\end{equation}
raises indices. For instance, the dual vectors
\begin{equation}
    e^i = g^{ij} e_j = h_i^{-2} e_i \quad \text{satisfy} \quad e^i \cdot e_k = \delta_k^i. \tag{11.156}
\end{equation}

The invariant squared distance $dp^2$ between nearby points (11.143) is
\begin{equation}
    dp^2 = \int d^m p = g_{ij} dx^i dx^j = h_i^2 (dx^i)^2 \tag{11.157}
\end{equation}
and the invariant volume element is
\begin{equation}
    dV = d^n p = h_1 \cdots h_n dx^1 \wedge \cdots \wedge dx^n = g dx^1 \wedge \cdots \wedge dx^n = g d^n x \tag{11.158}
\end{equation}
in which $g = \sqrt{\det g_{ij}}$ is the square-root of the positive determinant of $g_{ij}$.

The important special case in which all the scale factors $h_i$ are unity is cartesian coordinates in euclidean space (section 11.5).

We also can use basis vectors $\hat{e}_i$ that are orthonormal. By (11.154 & 11.156), these vectors
\begin{equation}
    \hat{e}_i = e_i / h_i = h_i e^i \quad \text{satisfy} \quad \hat{e}_i \cdot \hat{e}_j = \delta_{ij}. \tag{11.159}
\end{equation}

In terms of them, a physical and invariant vector $V$ takes the form
\begin{equation}
    V = e_i V^i = h_i \hat{e}_i V^i = e^i V_i = h_i^{-1} \hat{e}_i V_i = \hat{e}_i \nabla_i \tag{11.160}
\end{equation}
where
\begin{equation}
    \nabla_i \equiv h_i V^i = h_i^{-1} V_i \quad \text{(no sum)} \tag{11.161}
\end{equation}
The dot-product is then
\begin{equation}
    V \cdot U = g_{ij} V^i U^j = \nabla_i \overline{U}_i. \tag{11.162}
\end{equation}

In euclidian $n$-space, we even can choose coordinates $x^i$ so that the vectors $e_i$ defined by $dp = e_i dx^i$ are orthonormal. The metric tensor is then the $n \times n$ identity matrix $g_{ik} = e_i \cdot e_k = I_{ik} = \delta_{ik}$. But since this is euclidian $n$-space, we also can expand the $n$ fixed orthonormal cartesian unit vectors $\hat{\ell}$ in terms of the $e_i(x)$ which vary with the coordinates as $\hat{\ell} = e_i(x)(e_i(x) \cdot \hat{\ell})$. 