and that the real part of the forward scattering amplitude is given by the Kramers-Kronig integral (5.276) of the total cross-section

$$\mathrm{Re}(f(\omega)) = \frac{\omega^2}{2\pi^2 c} P \int_0^\infty \frac{\sigma_{\mathrm{tot}}(\omega')\, d\omega'}{\omega'^2 - \omega^2}. \tag{5.286}$$

So the real part of the index of refraction is

$$n_r(\omega) = 1 + \frac{cN}{\pi} P \int_0^\infty \frac{\sigma_{\mathrm{tot}}(\omega')\, d\omega'}{\omega'^2 - \omega^2}. \tag{5.287}$$

If the amplitude for forward scattering is of the Breit-Wigner form

$$f(\omega) = f_0 \frac{\Gamma/2}{\omega_0 - \omega - i\Gamma/2} \tag{5.288}$$

then by (5.285) the real part of the index of refraction is

$$n_r(\omega) = 1 + \frac{\pi c^2 N f_0 \Gamma(\omega_0 - \omega)}{\omega^2 \left[(\omega - \omega_0)^2 + \Gamma^2/4\right]} \tag{5.289}$$

and by (5.283) the group velocity is

$$v_g = c \left[1 + \frac{\pi c^2 N f_0 \Gamma\, \omega_0}{\omega^2} \frac{\left[(\omega - \omega_0)^2 - \Gamma^2/4\right]}{\left[(\omega - \omega_0)^2 + \Gamma^2/4\right]^2} \right]^{-1}. \tag{5.290}$$

This group velocity v_g is less than c whenever $(\omega - \omega_0)^2 > \Gamma^2/4$. But we get fast light $v_g > c$, if $(\omega - \omega_0)^2 < \Gamma^2/4$, and even backwards light, $v_g < 0$, if $\omega \approx \omega_0$ with $4\pi c^2 N f_0/\Gamma\omega_0 \gg 1$. Robert W. Boyd's papers explain how to make slow and fast light (Bigelow et al., 2003) and backwards light (Gehring et al., 2006).

We can use the principal-part identity (5.224) to subtract

$$0 = \frac{cN}{\pi} P \int_0^\infty \frac{\sigma_{\mathrm{tot}}(\omega)}{\omega'^2 - \omega^2}\, d\omega' \tag{5.291}$$

from the Kramers-Kronig integral (5.287) so as to write the index of refraction in the regularized form

$$n_r(\omega) = 1 + \frac{cN}{\pi} P \int_0^\infty \frac{\sigma_{\mathrm{tot}}(\omega') - \sigma_{\mathrm{tot}}(\omega)}{\omega'^2 - \omega^2}\, d\omega' \tag{5.292}$$

which we can differentiate and use in the group-velocity formula (5.283)

$$v_g(\omega) = c \left[1 + \frac{cN}{\pi} P \int_0^\infty \frac{\left[\sigma_{\mathrm{tot}}(\omega') - \sigma_{\mathrm{tot}}(\omega)\right](\omega'^2 + \omega^2)}{(\omega'^2 - \omega^2)^2}\, d\omega' \right]^{-1}. \tag{5.293}$$