in the lower half plane. The delta function in the second integral then gives $\pi/2$, so that
\[
I = \int dk \frac{e^{ik}}{2i(k + i\epsilon)} + \frac{\pi}{2} = \frac{\pi}{2}
\]
(5.229) as stated in (3.109).

Example 5.36 (The Feynman Propagator) Adding $\pm i\epsilon$ to the denominator of a pole term of an integral formula for a function $f(x)$ can slightly shift the pole into the upper or lower half plane, causing the pole to contribute if a ghost contour goes around the upper half-plane or the lower half-plane. Such an $i\epsilon$ can impose a boundary condition on a Green’s function.

The Feynman propagator $\Delta_F(x)$ is a Green’s function for the Klein-Gordon differential operator (Weinberg, 1995, pp. 274–280)
\[
(m^2 - \Box)\Delta_F(x) = \delta^4(x)
\]
(5.230) in which $x = (x^0, \mathbf{x})$ and
\[
\Box = \nabla^2 = -\frac{\partial^2}{\partial(x^0)^2}
\]
(5.231) is the four-dimensional version of the laplacian $\Delta \equiv \nabla \cdot \nabla$. Here $\delta^4(x)$ is the four-dimensional Dirac delta function (3.36)
\[
\delta^4(x) = \int \frac{d^4q}{(2\pi)^4} \exp[i(q \cdot x - q^0 x^0)] = \int \frac{d^4q}{(2\pi)^4} e^{iqx}
\]
(5.232) in which $qx = q \cdot x - q^0 x^0$ is the Lorentz-invariant inner product of the 4-vectors $q$ and $x$. There are many Green’s functions that satisfy Eq.(5.230). Feynman’s propagator $\Delta_F(x)$ is the one that satisfies boundary conditions that will become evident when we analyze the effect of its $i\epsilon$

\[
\Delta_F(x) = \int \frac{d^4q}{(2\pi)^4} \frac{\exp(iqx)}{q^2 + m^2 - i\epsilon} = \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{dq^0}{2\pi} \frac{e^{iq \cdot x - iq^0 x^0}}{q^2 + m^2 - i\epsilon}.
\]
(5.233)

The quantity $E_q = \sqrt{q^2 + m^2}$ is the energy of a particle of mass $m$ and momentum $q$ in natural units with the speed of light $c = 1$. Using this abbreviation and setting $\epsilon' = \epsilon/2E_q$, we may write the denominator as
\[
q^2 + m^2 - i\epsilon = q \cdot q - (q^0)^2 + m^2 - i\epsilon = (E_q - i\epsilon' - q^0) (E_q - i\epsilon' + q^0) + \epsilon'^2
\]
(5.234)