which satisfies the condition (4.112) that defines an asymptotic series

$$\begin{aligned}\lim_{x\to\infty} x^n R_n(x) &= \lim_{x\to\infty} (-1)^n \frac{n!\,e^{-x}}{x} \int_0^\infty e^{-u} \frac{du}{\left(1+\frac{u}{x}\right)^{n+1}} \\ &= \lim_{x\to\infty} (-1)^n \frac{n!\,e^{-x}}{x} \int_0^\infty e^{-u}\, du \\ &= \lim_{x\to\infty} (-1)^n \frac{n!\,e^{-x}}{x} = 0 \end{aligned} \quad (4.123)$$

for fixed n. □

Asymptotic series often occur in physics. In such physical problems, a small parameter λ usually plays the role of $1/x$. A perturbative series

$$S_n(\lambda) = \sum_{k=0}^{n} a_k\, \lambda^k \quad (4.124)$$

is an asymptotic expansion of the physical quantity $S(\lambda)$ if the remainder

$$R_n(\lambda) = S(\lambda) - S_n(\lambda) \quad (4.125)$$

satisfies for fixed n

$$\lim_{\lambda\to 0} \lambda^{-n}\, R_n(\lambda) = 0. \quad (4.126)$$

The WKB approximation and the Dyson series for quantum electrodynamics are asymptotic expansions in this sense.

4.13 Some Electrostatic Problems

Gauss's law $\nabla \cdot \boldsymbol{D} = \rho$ equates the divergence of the **electric displacement $\boldsymbol{D}$** to the density ρ of **free charges** (charges that are free to move in or out of the dielectric medium—as opposed to those that are part of the medium and bound to it by molecular forces). In electrostatic problems, Maxwell's equations reduce to Gauss's law and the static form $\nabla \times \mathbf{E} = 0$ of Faraday's law which implies that the electric field $\boldsymbol{E}$ is the gradient of an electrostatic potential $\boldsymbol{E} = -\nabla V$. (James Maxwell 1831–1879, Michael Faraday 1791–1867)

Across an interface with normal vector $\hat{\boldsymbol{n}}$ between two dielectrics, the tangential electric field is continuous while the normal electric displacement jumps by the surface density of free charge σ

$$\hat{\boldsymbol{n}} \times (\boldsymbol{E}_2 - \boldsymbol{E}_1) = 0 \quad \text{and} \quad \sigma = \hat{\boldsymbol{n}} \cdot (\boldsymbol{D}_2 - \boldsymbol{D}_1)\,. \quad (4.127)$$

In a **linear dielectric**, the electric displacement $\boldsymbol{D}$ is proportional to the