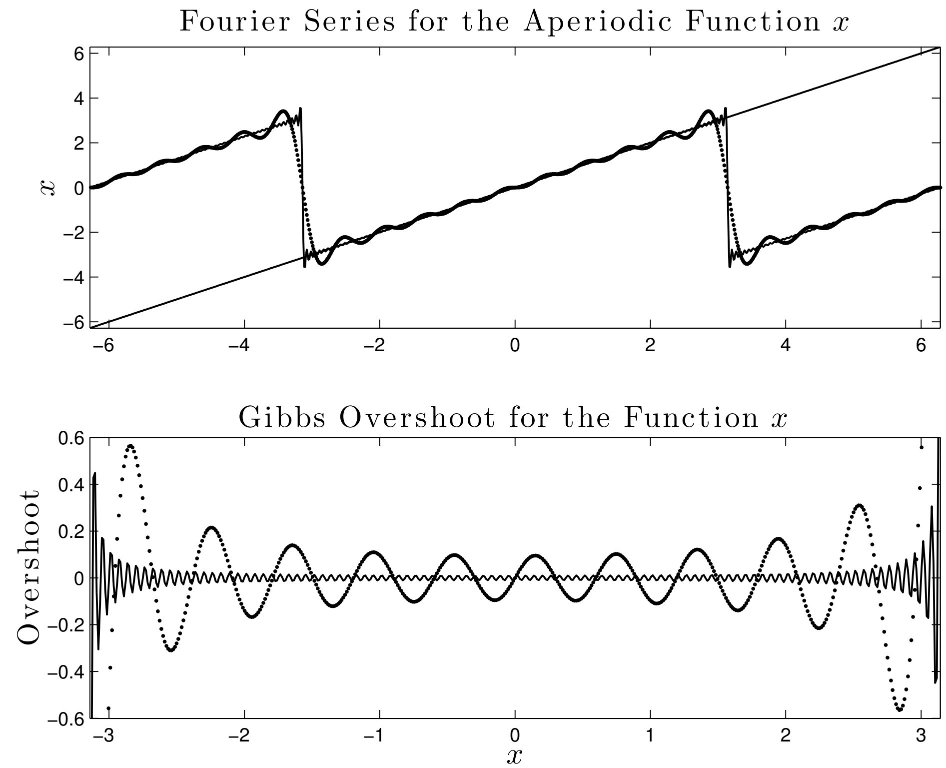

Figure 2.4 (top) The Fourier series (2.32) for the function x (solid line) with 10 terms (dots) and 100 terms (solid curve) for $-2\pi < x < 2\pi$. The Fourier series is periodic, but the function x is not. (bottom) The differences between x and the 10-term (dots) and the 100-term (solid curve) on $(-\pi, \pi)$ exhibit a Gibbs overshoot of about 9% at $x \gtrsim -\pi$ and at $x \lesssim \pi$.

2.5 Stretched Intervals

If the interval of periodicity is of length L instead of 2π, then we may use the phases $\exp(i2\pi nx/\sqrt{L})$ which are orthonormal on the interval $[0, L]$

$$\int_0^L dx \left(\frac{e^{i2\pi nx/L}}{\sqrt{L}}\right)^* \frac{e^{i2\pi mx/L}}{\sqrt{L}} = \delta_{nm}. \tag{2.33}$$

The Fourier series

$$f(x) = \sum_{n=-\infty}^{\infty} f_n \frac{e^{i2\pi nx/L}}{\sqrt{L}} \tag{2.34}$$