Solution to second homework problem.

Physics 524, April 23, 2015

Problem: Show that

\[
\ell E \ell / D \ell E = \left[ \frac{1}{2} (1 + \gamma_5) E \right] \dagger i \gamma^0 \ell \gamma^1 \frac{1}{2} (1 + \gamma_5) E = E \ell \gamma^0 \frac{1}{2} (1 + \gamma_5) E
\]  

(1)

where in Weinberg’s notation

\[
\bar{\psi} = \psi^\dagger i \gamma^0 = \psi^\dagger \beta = \psi^\dagger \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \quad \text{and} \quad \gamma_5 = \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]  

(2)

in which \( I \) is the \( 2 \times 2 \) identity matrix.

Solution: First, note that

\[
\frac{1}{2} (1 + \gamma^5) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = P_\ell
\]  

(3)

is a projection operator

\[
P_\ell^2 = P_\ell
\]  

(4)

on the left-handed two-component spinor

\[
\frac{1}{2} (1 + \gamma^5) E = E_\ell.
\]  

(5)

Thus, we have

\[
\bar{E}_\ell \psi E_\ell = \left[ \frac{1}{2} (1 + \gamma_5) E \right] \dagger i \gamma^0 \ell \gamma^1 \frac{1}{2} (1 + \gamma_5) E.
\]  

(6)

Next, we note that

\[
\left[ \frac{1}{2} (1 + \gamma_5) E \right] \dagger = E\dagger \frac{1}{2} (1 + \gamma_5).
\]  

(7)
and so

\[ [\frac{1}{2}(1 + \gamma_5)E]^\dagger i\gamma^0 \slashed{D}^{\ell} \frac{1}{2}(1 + \gamma_5)E = E^\dagger \frac{1}{2}(1 + \gamma_5)i\gamma^0 \slashed{D}^{\ell} \frac{1}{2}(1 + \gamma_5)E \]

\[ = E^\dagger i\gamma^0 \slashed{D}^{\ell} \frac{1}{2}(1 + \gamma_5)\frac{1}{2}(1 + \gamma_5)E \]

\[ = \bar{E}_\ell \slashed{D}^{\ell} \frac{1}{2}(1 + \gamma_5)E = \bar{E}_\ell \slashed{D}^{\ell} E_\ell. \]