Homework 8

(1) Use the fact that Pauli’s matrices satisfy
\[ \sigma_i \sigma_j = \delta_{ij} + i \epsilon_{ijk} \sigma_k \]  
(1)
to show that the Dirac matrices chosen as
\[ \gamma^0 = -i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \gamma = -i \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix} \]  
(2)
obey the he anticommutation relation
\[ \{ \gamma^a, \gamma^b \} \equiv \gamma^a \gamma^b + \gamma^b \gamma^a = 2 \eta^{ab} I \]  
(3)
in which \( I \) is the 4 \( \times \) 4 identity matirx and \( \eta \) is the 4 \( \times \) 4 diagonal matrix with \( \eta^{00} = -1 \) and \( \eta^{jj} = 1 \) for \( j = 1, 2, \) and 3. That is
\[ \eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \]  
(4)

(2) The fancy generators \( J^{ab} \) of the Lorentz group are related to the simple generators \( J \) and \( K \) of the Lorentz group by
\[ J^{ij} = \epsilon_{ijk} J_k \quad \text{and} \quad J^{0j} = K_j. \]  
(5)
We can write the fancy generators \( J^{ab} \) as commutators of gamma matrices
\[ J^{ab} = -\frac{i}{4} [\gamma^a, \gamma^b]. \]  
(6)
Express the simple generators in terms of the gamma matrices.

(3) Find explicit formulas for the simple generators of the Lorentz group \( J \) and \( K \). You can use a 2 \( \times \) 2 notation in which the Pauli matrices and the 2 \( \times \) 2 identity matrix are the entries. For instance, you should find
\[ J_3 = \frac{1}{2} \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}. \]  
(7)
Tentatively, this homework is due on 14 xi 2012.