Homework 8

(1) Use the fact that Pauli's matrices satisfy

$$\sigma_i \sigma_j = \delta_{ij} + i\epsilon_{ijk}\sigma_k \tag{1}$$

to show that the Dirac matrices chosen as

$$\gamma^0 = -i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \boldsymbol{\gamma} = -i \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix} \tag{2}$$

obey the he anticommutation relation

$$\{\gamma^a, \gamma^b\} \equiv \gamma^a \gamma^b + \gamma^b \gamma^a = 2\eta^{ab} I \tag{3}$$

in which I is the 4×4 identity matirx and η is the 4×4 diagonal matrix with $\eta^{00} = -1$ and $\eta^{jj} = 1$ for $j = 1$, 2, and 3. That is

$$\eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{4}$$

(2) The fancy generators J^{ab} of the Lorentz group are related to the simple generators $\boldsymbol{J}$ and $\boldsymbol{K}$ of the Lorentz group by

$$J^{ij} = \epsilon_{ijk} J_k \quad \text{and} \quad J^{0j} = K_j. \tag{5}$$

We can write the fancy generators J^{ab} as commutators of gamma matrices

$$J^{ab} = -\frac{i}{4}[\gamma^a, \gamma^b]. \tag{6}$$

Express the simple generators in terms of the gamma matrices.

(3) Find explicit formulas for the simple generators of the Lorentz group $\boldsymbol{J}$ and $\boldsymbol{K}$. You can use a 2×2 notation in which the Pauli matrices and the 2×2 identity matrix are the entries. For instance, you should find

$$J_3 = \frac{1}{2} \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}. \tag{7}$$

Tentatively, this homework is due on 14 xi 2012.