Homework 6

Show that the three 3×3 matrices $t_a = i\omega_a$ where

$$\omega_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \omega_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad \omega_3 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (1)$$

obey the commutation relations of the rotation group

$$[t_a, t_b] = i\, f_{abc}\, t_c \qquad (2)$$

in which the structure constants are given by the Levi-Civita symbol ϵ_{abc}

$$f_{abc} = \epsilon_{abc} \qquad (3)$$

so that

$$[t_a, t_b] = i\, \epsilon_{abc}\, t_c. \qquad (4)$$

The Levi-Civita symbol ϵ_{abc} is totally antisymmetric with $\epsilon_{123} = 1$.
    This short homework is due on 31 x 2012.