

First Set of Homework Problems

Problem 1: Path integrals in real time represent the time-evolution operator $\exp(-itH)$. In class we saw that in the limit $\epsilon \rightarrow 0$

$$\begin{aligned}
 \langle q'' | e^{-i\epsilon H} | q' \rangle &= \int_{-\infty}^{\infty} \langle q'' | \exp \left[-i\epsilon \frac{p^2}{2m} \right] | p' \rangle \langle p' | \exp [-i\epsilon V(q)] | q' \rangle dp' \\
 &= \frac{1}{2\pi} e^{-i\epsilon V(q')} \int_{-\infty}^{\infty} \exp \left[-i\epsilon \frac{p'^2}{2m} + i p' (q'' - q') \right] dp' \\
 &= \frac{1}{2\pi} e^{-i\epsilon V(q')} \int_{-\infty}^{\infty} \exp \left[-i\epsilon \frac{p'^2}{2m} + i\epsilon p' \dot{q}' \right] dp' \\
 &= \left(\frac{m}{2\pi i\epsilon} \right)^{1/2} \exp \left[i\epsilon \left(\frac{m \dot{q}'^2}{2} - V(q') \right) \right]. \tag{1}
 \end{aligned}$$

When we link together n of these matrix elements, we find

$$\langle q_n | e^{-in\epsilon H} | q_0 \rangle = \left(\frac{m}{2\pi i\epsilon} \right)^{n/2} \iiint_{-\infty}^{\infty} \exp \left\{ i\epsilon \sum_{j=0}^{n-1} \left[\frac{1}{2} m \dot{q}_j^2 - V(q_j) \right] \right\} dq_1 \dots dq_{n-1}. \tag{2}$$

Your task is to set $n = 3$ and $V(q) = 0$ in this formula and to carry out the integrations over q_1 and q_2 explicitly as I did in class for the Boltzmann operator $e^{-\beta H}$.

Problem 2: Do the first problem again but with $V(q) = \frac{1}{2}m\omega^2 q^2$.

This homework will be due not before next Wednesday, 5 September.