$$\begin{aligned} \langle q_3|e^{-3i\epsilon H}|q_0\rangle &= \iint_{-\infty}^{\infty} \langle q_3|e^{-i\epsilon H}|q_2\rangle\langle q_2|e^{-i\epsilon H}|q_1\rangle\langle q_1|e^{-i\epsilon H}|q_0\rangle \, dq_1 dq_2 \qquad (1) \\ &= \left(\frac{m}{2\pi i\epsilon}\right)^{3/2} \iint_{-\infty}^{\infty} \exp\left\{ i\epsilon \sum_{j=0}^{2} \left[\, \tfrac{1}{2}\, m\, \dot{q}_j^2 - V(q_j) \,\right] \right\} dq_1 dq_2 . \end{aligned}$$

Let's ignore the potential $V(q)$ and integrate over q_1

$$\begin{aligned} I_1 &= \left(\frac{m}{2\pi i\epsilon}\right)^{1/2} \int_{-\infty}^{\infty} \exp\left\{ i\frac{m}{2\epsilon}\left[(q_2-q_1)^2 + (q_1-q_0)^2\right]\right\} dq_1 \\ &= \left(\frac{m}{2\pi i\epsilon}\right)^{1/2} \int_{-\infty}^{\infty} \exp\left\{ i\frac{m}{\epsilon}\left[q_1^2 - (q_2+q_0)q_1 + \tfrac{1}{2}(q_2^2+q_0^2)\right]\right\} dq_1 . \end{aligned} \qquad (2)$$

Using the formula

$$\int_{-\infty}^{\infty} \exp\left(-iax^2 + ibx\right) \, dx = \sqrt{\frac{\pi}{ia}} \, \exp\left(i\frac{b^2}{4a}\right) \qquad (3)$$

with $a = -m/\epsilon$ and $b = -m(q_2+q_0)/\epsilon$, we get

$$\begin{aligned} I_1 &= \left(\frac{m}{2\pi i\epsilon}\right)^{1/2} \sqrt{\frac{i\pi\epsilon}{m}} \exp\left[-im(q_2+q_0)^2/4\epsilon + im(q_2^2+q_0^2)/2\epsilon\right] \\ &= \frac{1}{\sqrt{2}} e^{im(q_2-q_0)^2/4\epsilon} . \end{aligned} \qquad (4)$$

The amplitude now is

$$\begin{aligned} \langle q_3|e^{-3i\epsilon H}|q_0\rangle &= \frac{m}{2\sqrt{2}\pi i\epsilon} \int_{-\infty}^{\infty} \exp\left[i\frac{m}{2\epsilon}\left[(q_3-q_2)^2 + \tfrac{1}{2}(q_2-q_0)^2\right]\right] dq_2 \\ &= \frac{m}{2\sqrt{2}\pi i\epsilon} \int_{-\infty}^{\infty} \exp\left[i\frac{m}{2\epsilon}\left(\frac{3}{2}q_2^2 - (2q_3+q_0)q_2 + q_3^2 + \tfrac{1}{2}q_0^2\right)\right] dq_2 \end{aligned} \qquad (5)$$

Now we do the q_2 integral using (3) with $a = -3m/4\epsilon$ and $b = -m(2q_3 + q_0)/2\epsilon$. We find

$$\langle q_3|e^{-3i\epsilon H}|q_0\rangle = \left(\frac{m}{2\pi i 3\epsilon}\right)^{1/2} e^{im(q_3-q_0)^2/2(3\epsilon)} . \qquad (6)$$

We now leap from 3 to n and replace 3ϵ with $n\epsilon = t$ so as to find that

$$\langle q_\beta|e^{-itH}|q_0\rangle = \left(\frac{m}{2\pi i t}\right)^{1/2} e^{im(q_\beta-q_0)^2/2t} . \qquad (7)$$