10.38 Show that under the unitary Lorentz transformation (10.273), the action density (10.258) is Lorentz covariant (10.259).

10.39 Show that under the unitary Lorentz transformations (10.257 & 10.273), the Majorana mass terms (10.266 & 10.279) are Lorentz covariant.

Solution: Transposition flips the sign of $\sigma_2$ but leaves $\sigma_1$ and $\sigma_3$ invariant. And because $\sigma_2$ anti commutes with both $\sigma_1$ and $\sigma_3$, it follows that

$$\sigma_2 z \cdot \sigma^T \sigma_2 = - z \cdot \sigma. \quad (10.311)$$

Since $\sigma_2^2 = 1$, we have for any $2 \times 2$ matrix $A$, the relation $(\sigma_2 A \sigma_2)^2 = \sigma_2 A^2 \sigma_2$ as well as $(\sigma_2 A \sigma_2)^n = \sigma_2 A^n \sigma_2$. Thus

$$\sigma_2 e^A \sigma_2 = e^{\sigma_2 A \sigma_2}. \quad (10.312)$$

Thus under a Lorentz transformation $U(L) \xi^T(x) \sigma_2 \xi(x) = \left(e^{z \cdot \sigma/2} \xi(Lx)\right)^T \sigma_2 e^{z \cdot \sigma/2} \xi(Lx)$

$$= \xi(Lx)^T \left(e^{z \cdot \sigma/2}\right)^T \sigma_2 e^{z \cdot \sigma/2} \xi(Lx)
= \xi(Lx)^T \exp(z \cdot \sigma^T/2) \sigma_2 e^{z \cdot \sigma/2} \xi(Lx)
= \xi(Lx)^T \sigma_2 \sigma_2 \exp(z \cdot \sigma^T/2) \sigma_2 e^{z \cdot \sigma/2} \xi(Lx)
= \xi(Lx)^T \sigma_2 \exp(z \cdot \sigma^T/2) \sigma_2 \sigma_2 e^{z \cdot \sigma/2} \xi(Lx)
= \xi(Lx)^T \sigma_2 \exp(-z \cdot \sigma/2) e^{z \cdot \sigma/2} \xi(Lx)
= \xi(Lx)^T \sigma_2 e^{-z \cdot \sigma/2} e^{z \cdot \sigma/2} \xi(Lx)
= \xi(Lx)^T \sigma_2 \xi(Lx). \quad (10.313)$$

10.40 Show that the definitions of the gamma matrices (10.281) and of the generators (10.283) imply that the gamma matrices transform as a 4-vector under Lorentz transformations (10.284).

10.41 Show that (10.283) and (10.284) imply that the generators $J^{ab}$ satisfy the commutation relations of the Lorentz group.

10.42 Show that the spinor $\zeta = \sigma_2 \xi^*$ defined by (10.295) is right handed (10.273) if $\xi$ is left handed (10.257).

10.43 Use (10.303) to get (10.304 & 10.305).