$$Q_a = \psi_\alpha(\vec{x}, t)$$

$$P_a = i\,\psi_\alpha^\dagger(\vec{x}, t)$$

$$\bar{\psi} = \psi^\dagger \beta = i\psi^\dagger \gamma^0$$

so

$$P_a = -\bar{\psi}\gamma^0$$

$$\xi_1 = \begin{pmatrix} 0 & 0 & & \\ 1 & 0 & & \\ & & 0 & 0 \\ & & 1 & 0 \end{pmatrix} \qquad \xi_2 = \begin{pmatrix} & & & \\ & & & \\ 1 & 0 & & \\ 0 & -1 & & \end{pmatrix}$$

$$\{\xi_1, \xi_2\} = 0 \qquad \xi_1^2 = \xi_2^2 = 0$$

$$\xi_1 \cdots \xi_n \longleftrightarrow 2^n \times 2^n \text{ matrices}$$

$$f(\xi) = \left(\prod \xi_n\right) c$$

$$\prod \xi_n' = (\operatorname{Det} S) \prod \xi_n$$

So

$$f(\xi) = (\operatorname{Det} S)^{-1} \prod \xi_n' \, c.$$

So

$$\int \hat{\prod} d\xi_n' f(\xi) = \int \hat{\prod} d\xi_n' (\operatorname{Det} S)^{-1} \prod \xi_n' \, c$$

$$= (\operatorname{Det} S)^{-1} \int \hat{\prod} d\xi_n' \prod \xi_n' \, c$$

$$= (\operatorname{Det} S)^{-1} c$$

$$= (\operatorname{Det} S)^{-1} \int \hat{\prod} d\xi_n \prod \xi_n \, c$$

$$= (\operatorname{Det} S)^{-1} \int \hat{\prod} d\xi_n \, f(\xi).$$

$$\mathcal{J}(f,g) = \int dq' dp' \exp\Big[-i \int dx\, y \big[p'_m - \int g_n \mathcal{D}^{-1}_{ny\, mx}\big] \mathcal{D}_{mx\, n'y'}$$

$$\big[q'_{n'}(y') - \int dx' \mathcal{D}^{-1}_{n'y'\, m'x'} f\big]$$

$$-i \int d^4x \big[p'_m(x) - \int dy\, g_n(y) \mathcal{D}_{ny\, mx}\big] f_m(x)$$

$$-i \int d^4y\, g_n(y) \big[q'_n(y) - \int d^4x\, \mathcal{D}_{ny\, mx} f_m x\big] \Big]$$

$$= \int dq' dp' \exp\Big\{ -i \int d^4x d^4y\, p'_m \mathcal{D}_{mx\, ny}\, q'_n(y)$$

$$+i \int dx dy\, p'_m(x) \mathcal{D}_{mx\, ny} \mathcal{D}^{-1}_{ny\, m'x'} f_{m'x'}$$

$$+i \int dx dy \int g_n \mathcal{D}^{-1} \mathcal{D}\, q'_n$$

$$-i \int dx dy\, g_n \mathcal{D}^{-1} \mathcal{D} \mathcal{D}^{-1} f$$

$$-i \int p' f \quad -i\, g q' \quad +2i \int dx\, g \mathcal{D}^{-1} f$$

$$= e^{i \int g \mathcal{D}^{-1} f} \int e^{-i \int p' \mathcal{D} q'} \quad \boxed{\pm i g q' + \underline{i}\, p' f} = 0$$

So

$$\mathcal{J}(f,g) = \int \Pi\, dq\, dp\, e^{-i\sum_{mn}\int d^4x\, d^4y\; p_m(x)\, D_{mx,ny}\, q_n(y)}$$

$$\times\, e^{-i\int d^4x \sum_m p_m(x) f_m(x) - i\sum_n \int d^4y\, g_n(y)\, q_n(y)}$$

$$= \eta\; e^{i\sum_{mn}\int d^4x\, d^4y\; g_n(y)\, D^{-1}_{ny\,mx}\, f_m(x)}$$

where η is a number independent of f & g.

So

$$\int \Pi\, dq\, dp\; e^{-i\sum\int d^4x' d^4y'\; p_m(x')\, D_{mx'ny'}\, q_n(y')}\; p_r(x)\, q_s(y)$$

$$= i^2 \frac{\delta^2 \mathcal{J}(f,g)}{\delta f_r(x)\,\delta g_s(y)}\Bigg|_{f=g=0}$$

$$= \eta\, i^2 \frac{\delta^2}{\delta f_r(x)\,\delta g_s(y)}\; e^{i\sum_{mn}\int d^4x\, d^4y\; g_n(y)\, D^{-1}_{ny\,mx}\, f_m(x)}\Bigg|_{f=g=0}$$

$$= \eta(-)\frac{\delta}{\delta f_r(x)}\sum_m i\int d^4x\, D^{-1}_{sy\,mx}\, f_m(x)\Bigg|_{f=0}$$

$$= \eta\,(-i)\, D^{-1}_{sy\,rx}$$

So $\quad -i\Sigma\int d^4x\, d^4y\; p(x)\, \mathcal{D}\, q(y)$

$\int \Pi\, dq\, dp\; e \qquad p_r(x)\; q_s(y)\; p_m(x')\; q_n(y')$

$$= i^4 \frac{\delta^4\, \mathcal{J}(f,g)}{\delta f_r(x)\,\delta g_s(y)\,\delta f_m(x')\,\delta g_n(y')}\Bigg|_{f=g=0}$$

$$= \eta\, i^4 \frac{\delta^4}{\delta f_r(x)\,\delta g_s(y)\,\delta f_m(x')\,\delta g_n(y')}\, e^{i\sum_{m'n'}\int d^4x''\, d^4y''\, g_{n'}(y'')\, \mathcal{D}^{-1}_{n'y''m'x''}\, f_{m'}(x'')}\Bigg|_{f=g=0}$$

$$= \eta\, i^4 \frac{\delta^3}{\delta f_r(x)\,\delta g_s(y)\,\delta f_m(x')}\; i\int d^4x''\, \mathcal{D}^{-1}_{ny'm'x''}\, f_{m'}(x'')\Big)\, e^{i\Sigma\int g\mathcal{D}^{-1}f}\Bigg|_{f=g=0}$$

$$= \eta\, i^4 \frac{\delta^2}{\delta f_r(x)\,\delta g_s(y)}\Big[i\mathcal{D}^{-1}_{ny'mx'} - \int d^4x''\, \mathcal{D}^{-1}_{ny'm'x''}\, f_{m'}(x'') \times \int d^4y''\, g_{n'}(y'')\, \mathcal{D}^{-1}_{n'y''mx'}\Big]\, e^{i\Sigma\int g\mathcal{D}^{-1}f}\Bigg|_0$$

$$= \eta\, i^4 \frac{\delta}{\delta f_r(x)}\Big[-\mathcal{D}^{-1}_{ny'mx'}\int \mathcal{D}^{-1}_{sym'x''}\, f_{m'}(x'')\, d^4x'' + \int d^4x''\, \mathcal{D}^{-1}_{ny'm'x''}\, f_{m'}(x'')\, \mathcal{D}^{-1}_{symx'}\Big]\, e^{i\Sigma\int g\mathcal{D}^{-1}f}\Bigg|_{f,g=0}$$

$$= \eta i^4 \Big[- \mathcal{D}^{-1}_{ny'mx'} \mathcal{D}^{-1}_{5y\,rx}$$

$$+ \mathcal{D}^{-1}_{ny'rx} \mathcal{D}^{-1}_{5y\,mx'} \Big]$$

$$= \eta \Big[(-i \mathcal{D}^{-1}_{ny'mx'})(-i \mathcal{D}^{-1}_{5y\nu x})$$

$$- (-i \mathcal{D}^{-1}_{ny'\nu x})(-i \mathcal{D}^{-1}_{5y\,mx'}) \Big]$$

and we ignore η.

Now

$$\int d^4x \; p\dot{q} - H_0 = - \int d^4x \; \bar{\psi} \; [\gamma^\mu \partial_\mu + m] \psi$$

$$= - \int d^4x d^4y \; p \, \mathcal{D} \, q \qquad p = -\bar{\psi}\gamma^0, \quad q = \psi$$

$$= - \int d^4x d^4y \; -\bar{\psi}\gamma^0 \mathcal{D} \psi$$

So

$$-\gamma^0 \mathcal{D} = [\not\partial + m]$$

$$\mathcal{D} = \gamma^0 [\not\partial + m] \qquad \gamma^{0^2} = -1$$