$$\mathcal{L} = -\frac{1}{4} F_b^{\mu\nu} F_{\mu\nu b} - \bar{\Psi}_b [\gamma_\mu(\partial^\mu + i e_b A_b^\mu) + m_b] \Psi_b$$

$$\Psi_b = \sqrt{Z_2}\,\Psi \qquad A_b^\mu = \sqrt{Z_3}\,A^\mu \qquad e_b = e/\sqrt{Z_3}$$

$$m_b = m - \delta m$$

$$\mathcal{L} = -\frac{Z_3}{4} F^{\mu\nu} F_{\mu\nu} - Z_2 \bar{\Psi} [\gamma_\mu(\partial^\mu + i e A^\mu) + m - \delta m] \Psi$$

$$= -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \bar{\Psi} [\gamma_\mu(\partial^\mu + i e A^\mu) + m] \Psi$$

$$-\frac{(Z_3 - 1)}{4} F^{\mu\nu} F_{\mu\nu} - (Z_2 - 1) \bar{\Psi} [\gamma_\mu(\partial^\mu + i e A^\mu) + m] \Psi$$

$$+ Z_2 \delta m \bar{\Psi} \Psi$$

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2$$

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\Psi} [\gamma_\mu \partial^\mu + m] \Psi$$

$$\mathcal{L}_1 = -ie \bar{\Psi} \gamma_\mu A^\mu \Psi$$

$$\mathcal{L}_2 = -\frac{Z_3 - 1}{4} F^{\mu\nu} F_{\mu\nu} - (Z_2 - 1) \bar{\Psi} [\gamma_\mu \partial^\mu + m] \Psi$$

$$- ie (Z_2 - 1) \bar{\Psi} A^\mu \gamma_\mu \Psi + Z_2 \delta m \bar{\Psi} \Psi$$

$$\Sigma^*(p) = \Sigma^*_{1loop} - (Z_2-1)(i\not p + m) + Z_2\delta m$$

$$= \Sigma^*_{1loop} - (Z_2-1)(i\not p+m) + \delta m + (Z_2-1)\delta m$$

if $(Z_2-1)\delta m$ is taken to be $O(e^4)$ and left out

$$2i(1-x)\not p + 4m = -2m(1-x) + 4m = 2m + 2mx$$
$$= 2m(1+x)$$

$$\delta m = -\Sigma^*_{1loop}\Big|_{p^2=-m^2}.$$

$$= \frac{2m\pi^2 e^2}{(2\pi)^4}\int_0^1 dx\,(1+x)\ln\left(\frac{m^2x^2+\mu^2(1-x)}{m^2X^2}\right) \quad (11.4.9)$$

Now
$$\Sigma^*(p) = \sum_{n=2}^{\infty}(i\not p+m)^n c_n \qquad \text{so}$$

$$\frac{d\Sigma^*(p)}{i\,d\not p} = \sum_{n=2}^{\infty} n(i\not p+m)^{n-1}c_n \qquad \text{so}$$

$$\frac{d\Sigma^*(p)}{i\,d\not p}\Bigg|_{i\not p=-m} = 0 \qquad \text{so we need}$$

$$0 = \frac{d\Sigma^*_{1loop}}{i\,d\not p}\Bigg|_{p^2=-m^2} - (Z_2-1) \qquad \text{or}$$

$$Z_2 - 1 = -i\,\frac{d\Sigma^*_{1loop}}{d\not p}\Bigg|_{p^2=-m^2}$$

Now $-\dfrac{dp^2}{di\not{p}} = 2i\not{p}$

because $(i\not{p})^2 = -p_\mu p_\nu \gamma^\mu \gamma^\nu = -p^2$.

So

$$\frac{d\,f(p^2)}{di\not{p}} = \frac{dp^2}{di\not{p}}\,\frac{df(p^2)}{dp^2}$$

$$= -2i\not{p}\,\frac{df(p^2)}{dp^2}.$$ So

$$\frac{d\,\Sigma^*_{1\,loop}}{di\not{p}} = \frac{-\pi^2 e^3}{(2\pi)^4}\int_0^1 dx\left\{2(1-x)\ln\left(\frac{p^2x(1-x)+m^2x+\mu^2(1-x)}{p^2x(1-x)+m^2x}\right)\right.$$

$$+\left[2i(1-x)\not{p}+4m\right](-2i\not{p})\,\frac{p^2x(1-x)+m^2x}{p^2x(1-x)+m^2x+\mu^2(1-x)}$$

$$\left.\cdot\left[\frac{x(1-x)}{p^2x(1-x)+m^2x}-\frac{\left[p^2x(1-x)+m^2x+\mu^2(1-x)\right]x(1-x)}{(p^2x(1-x)+m^2x)^2}\right]\right\}\Bigg|_{\substack{p^2=-m^2\\ i\not{p}=-m}}$$

$i\not{p}=-m$

$$\frac{d\Sigma^*_{1\,loop}}{d\not{p}} = -\frac{\pi^2 e^2}{(2\pi)^4}\int_0^1 dx \left\{ 2(1-x)\ln\left(\frac{m^2x^2+\mu^2(1-x)}{m^2x^2}\right)\right.$$

$$+\left(-4m^2(1-x)+8m^2\right)\left(\frac{m^2x^2}{m^2x^2+\mu^2(1-x)}\right)$$

$4m^2(1+x)$

$$\left.\cdot\left[\frac{x(1-x)}{m^2x^2} - \frac{[m^2x^2+\mu^2(1-x)]\,x(1-x)}{(m^2x^2)^2}\right]\right\}$$

$$= -\frac{\pi^2e^2}{(2\pi)^4}\int_0^1 dx\left\{2(1-x)\ln\left(\frac{m^2x^2+\mu^2(1-x)}{m^2x^2}\right)\right.$$

$$\left.+\frac{4m^4x^2(1+x)}{m^2x^2+\mu^2(1-x)}\left[\frac{-\mu^2(1-x)^2x}{(m^2x^2)^2}\right]\right\}$$

$$= -\frac{2\pi^2e^2}{(2\pi)^4}\int_0^1 dx\left\{(1-x)\ln\left(\frac{m^2x^2+\mu^2(1-x)}{m^2x^2}\right)\right.$$

$$\left.-\frac{2\mu^2(1-x)^2(1+x)}{x(m^2x^2+\mu^2(1-x))}\right\} = Z_2 - 1$$

(11.4.10)

$$8m^2-4m^2+4m^2x = 4m^2(1+x)$$

So

$$\Sigma^*(p) = \Sigma^*_{1\,loop}(p) - \Sigma^*_{1\,loop}(p^2 = m^2) - (i\not{p}+m)\,\Sigma'^*_{1\,loop}(p^2 = m^2)$$

$$= \frac{-\pi^2 e^2}{(2\pi)^4}\int_0^1 dx\,\left[2i(1-x)\not{p} + 4m\right] \ln\left(\frac{p^2 x(1-x) + m^2 x + \mu^2(1-x)}{p^2 x(1-x) + m^2 x}\right)$$

$$+ \frac{2m\pi^2 e^2}{(2\pi)^4}\int_0^1 dx\,(1+x)\ln\left(\frac{m^2x^2 + \mu^2(1-x)}{m^2x^2}\right)$$

$$+ (i\not{p}+m)\frac{2\pi^2 e^2}{(2\pi)^4}\int_0^1 dx\left\{(1-x)\ln\left(\frac{m^2x^2+\mu^2(1-x)}{m^2x^2}\right) - \frac{2\mu^2(1-x)^2(1+x)}{x\left(m^2x^2 + \mu^2(1-x)\right)}\right\}$$

$\mu^2 \to \infty$

$$\Sigma^*(p) = -\frac{2\pi^2 e^2}{(2\pi)^4}\int_0^1 dx\,\left[i(1-x)\not{p} + 2m\right]\ln\left(\frac{\mu^2(1-x)}{p^2 x(1-x) + m^2 x}\right)$$

$$+ \frac{2m\pi^2 e^2}{(2\pi)^4}\int_0^1 dx\,(1+x)\ln\left(\frac{\mu^2(1-x)}{m^2x^2}\right)$$

$$+ (i\not{p}+m)\frac{2\pi^2 e^2}{(2\pi)^4}\int_0^1 dx\,(1-x)\left[\ln\left(\frac{\mu^2(1-x)}{m^2x^2}\right) - \frac{2}{x}(1-x^2)\right]$$

The μ^2-part is

$$\frac{-2\pi^2 e^2}{(2\pi)^4}\int_0^1 dx\Big\{[\,i(1-x)\not p+2m]\ln\mu^2$$

$$-m(1+x)\ln\mu^2$$

$$-(i\not p+m)(1-x)\ln\mu^2\Big\}$$

$$=\frac{-2\pi^2e^2}{(2\pi)^4}\ln\mu^2\int_0^1 dx\,\big(2m-(1+x)m-(1-x)m\big)=0$$

and it cancels, leaving to this order

$$\Sigma^*(p)=-\frac{2\pi^2e^2}{(2\pi)^4}\int_0^1 dx\Big\{[\,i(1-x)\not p+2m]\ln\left(\frac{m^2(1-x)}{p^2x(1-x)+m^2x}\right)$$

$$-m(1+x)\ln\left(\frac{1-x}{x^2}\right)$$

$$-(i\not p+m)\left[(1-x)\ln\left(\frac{1-x}{x^2}\right)-\frac{2(1-x^2)}{x}\right]\Big\}$$

(11.4.14).