Superconductivity

Electrons form Cooper pairs which are bosons and which therefore can condense giving rise to superconductivity when the temperature drops enough.

Landau and Ginzburg let \( \phi(x) \) stand for the Cooper pairs. Here \( x \equiv x^2 \). One calls \( \phi(x) \) and \( M(x) \) order parameters.

Now \( \phi(x) \) is a 2- charges complex field, so \( \text{2D} \phi \rightarrow \text{Di} \phi \equiv \text{Di} \phi - i z e A \).

The free energy is

\[
\mathcal{F} = \frac{1}{4} F_{ij}^2 + \text{D} \phi |^2 + \alpha |\phi|^2 + \frac{1}{2} |\phi|^4 + \ldots
\]

which is \( U(1) \) invariant under \( \phi \rightarrow e^{i \varphi} \phi \) and \( A_i \rightarrow A_i + e \Lambda \).

Take \( b > 0 \) and \( \alpha(T) = \alpha(T - T_c) \), \( \alpha > 0 \)

5. For \( T < T_c \), \( \alpha(T) < 0 \), and \( |\phi|^2 \) gets a mean value when \( T < T_c \)

\[
0 = a + b |\phi|^2 \quad \Rightarrow \quad |\phi| = \sqrt{-\frac{a}{b}} \equiv v.
\]
Then for $T < T_c$ and $\phi = 0$

$$J = \frac{1}{4} \nabla \phi^2 + 1 2 \varepsilon \phi - i 2 e A \phi^1 + \cdots$$

$$= \frac{1}{2} \vec{B}^2 + (2e\nu)^2 A^2$$

so photon has mass $\frac{1}{2} M^2 = (2e\nu)^2$

$M^2 = 8(e\nu)^2$.

**Meissner Effect**

Below $T_c$ the superconductor excludes $\vec{B}$. Why? Because

$$\vec{B} = \nabla \times \vec{A}$$

So if $\vec{A} = B_0 \vec{n} \phi$, then

$$B_2 = B_0.$$

But then

$$F = \frac{1}{2} B_0^2 + (2e\nu) B_0^2 \nu^2$$

which grows quadratically with $\nu$.
So the superconductor excludes magnetic fields.
London's penetration depth

Suppose \( A = A_0 e^{-x/l} \).

Then \( B = \frac{A_0}{e} e^{-x/l} \) and

\[
F \sim \frac{1}{2} \frac{A_0^2}{l^2} e^{2x/l} + (2\pi e^2 \hbar^2) A_0 e^{-2x/l}
\]

\[
\int F dx = A_0^2 \left( \frac{1}{2e^2} + (2\pi e^2)^2 \right) \frac{1}{l} \alpha \frac{1}{2l} + (2\pi e^2)^2 l
\]

So \( 0 = -\frac{1}{2e^2} + (2\pi e^2)^2 \) or \( 2l^2 = \frac{1}{(2\pi e^2)^2} \)

So \( l = \frac{1}{\sqrt{8\pi e^2}} = \frac{1}{2\pi e^2} \)

\[
= \frac{1}{\sqrt{2} e} \sqrt{\frac{\hbar}{m}}
\]

To find the coherence length, we note that \( \hbar x = -\sqrt{\alpha} \) 1x1

\[
<\phi(x)\phi(0)> \sim \int \frac{d^3k e^{-i k x}}{\hbar^2 + a} \sim \frac{e}{4\pi l^2} \text{1x1}
\]

So the coherence length is \( l_{\rho} \sim \frac{1}{\sqrt{\alpha}} \).
The ratio of the two lengths is
\[ \frac{d_c}{d_\phi} \sim \frac{1}{\sqrt{b}} \sqrt{-a} \sim \frac{\sqrt{b}}{e} = \frac{\sqrt{-a}}{ev} \]

\[ = \frac{m_\phi}{m_A} \]

Julian Amolkin in class pointed out that the Cooper pairs can't easily radiate photons because the photons in a superconductor below $T_c$ are MASSIVE. Thus, the Cooper pairs superconduct.