Spinor renormalization

$$\mathcal{L} = -\bar{\psi}_b(\partial\!\!\!/ + m_b)\psi_b - V_b(\bar{\psi}_b\psi_b)$$

$$\psi_b = \sqrt{Z_2}\,\psi$$

$$m_b = m - \delta m$$

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$$

$$\mathcal{L}_0 = -\bar{\psi}(\partial\!\!\!/ + m)\psi$$

$$\mathcal{L}_1 = -(Z_2 - 1)\bar{\psi}(\partial\!\!\!/ + m)\psi + Z_2\,\delta m\,\bar{\psi}\psi - V_b(Z_2\bar{\psi}\psi).$$

The complete propagator is again a sum

$$S'(k) = S + S\Sigma^* S + S\Sigma^* S\Sigma^* S + \cdots$$

$$= S\,\frac{1}{1 - \Sigma^* S}.$$

Here the lowest-order propagator is

$$S(k) = \frac{-ik\!\!\!/ + m}{k^2 + m^2 - i\epsilon} = \frac{1}{ik\!\!\!/ + m - i\epsilon}$$

since

$$(-ik\!\!\!/ + m)(ik\!\!\!/ + m - i\epsilon) = k^2 + m^2 - im\epsilon - \epsilon k\!\!\!/ .$$

So

$$S'(k) = \frac{1}{ik\!\!\!/ + m - i\epsilon - \Sigma^*}$$ where again Σ^* is a

sum of all graphs  that can't be separated by one cut.

$$\Sigma^* = -(Z_2-1)(i\not{k}+m) + Z_2\delta m + \Sigma^*_{loop}(\not{k}).$$

So that S' will have a pole at $k^2=-m^2$ with unit residue we want

$$\Sigma^*(im) = 0$$

and

$$\left.\frac{\partial \Sigma^*(\not{k})}{\partial \not{k}}\right|_{\not{k}=im} = 0 .$$

Note $\not{k} = im \Rightarrow k^2 = -m^2.$

Also $S = \dfrac{1}{i\not{k}+m-i\epsilon}$ so $\not{k}=im$ is the pole.

$$0 = \Sigma^*(im) = -(Z_2-1)(-m+m) + Z_2\delta m + \Sigma^*_{loop}(im)$$

implies that

$$Z_2\delta m = -\Sigma^*_{loop}(im)$$

$$0 = \left.\frac{d\Sigma^*}{d\not{k}}\right|_{im} = -(Z_2-1)i + \frac{d\Sigma^*_{loop}}{d\not{k}}$$ gives us

$$Z_2 = 1 - i\left.\frac{\partial \Sigma^*_{loop}(\not{k})}{\partial \not{k}}\right|_{\not{k}=im}.$$

$S\Sigma^* \to 0$ as $\not{k} \to im$ so again we may ignore radiative corrections to external lines.