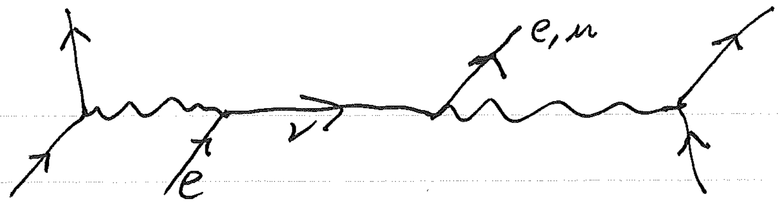
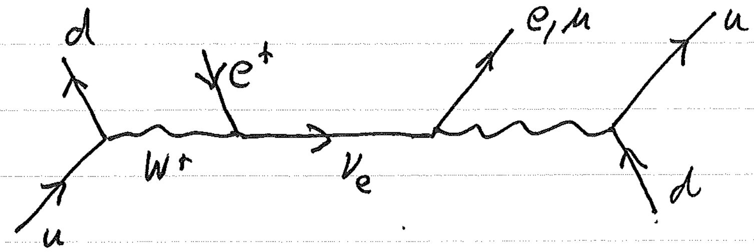
Informal notes on ν oscillations



or

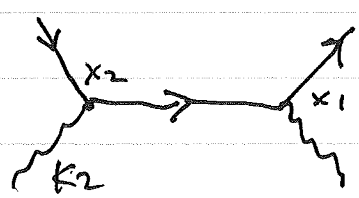


$$\langle 0 | T(\psi(x_1)\, \psi^\dagger(x_2)) | 0 \rangle \approx \theta(x_1^0 - x_2^0)\, \Delta_+(x_1 - x_2)$$

$$\Delta_+(x_1 - x_2) = \int \frac{d^3p}{(2\pi)^3\, 2p^0}\, e^{-ip(x_2 - x_1)}$$

Now x_2 is confined to some source region by some sort of gaussian. Likewise x_1 to a detector region. So we have something like

$$\int d^3x_1\, d^3x_2\, \frac{e^{-(x_2 - y_2)^2/2\sigma^2}}{(\sigma\sqrt{2\pi})^3}\, \frac{e^{-(x_1 - y_1)^2/2\sigma_2}}{(\sigma\sqrt{2\pi})^3}\, \frac{e^{-ip(x_2 - x_1)}}{(2\pi)^3\, 2p^0}\, d^3p$$



$$\times\, e^{-ik_2x_2 - ik_1x_1 + ip_2x_2 + ip_1x_1}$$

$$f(x_1^0, x_2^0)\, dx_1^0\, dx_2^0$$

~~call the~~ here $p(x_2 - x_1) = p^0(x_2^0 - x_1^0) - \vec{p} \cdot (\vec{x}_2 - \vec{x}_1)$

the d^3x_1 part is

$$\int d^3x_1 \, \frac{e^{-(x_1 - y_1)^2/2\sigma^2}}{(\sigma\sqrt{2\pi})^3} \, e^{-ix_1(p+p_1-k_1)}$$

$$= e^{-iy_1\cdot(p+p_1-k_1) - \frac{1}{2}\sigma^2(p+p_1-k_1)^2}$$

the x_2 part is

$$\int d^3x_2 \, \frac{e^{-(x_2 - y_2)^2/2\sigma^2}}{(\sigma\sqrt{2\pi})^3} \, e^{-ix_2(p_2-\vec{p}-k_2)}$$

$$= e^{-iy_2\cdot(p_2-p-k_2) - \frac{1}{2}\sigma^2(p_2-\vec{p}-k_2)^2}$$

So

$$\int \frac{d^3p}{(2\pi)^3 2p^0} f(x_1^0, x_2^0)\, dx_1^0\, dx_2^0 \, e^{-iy_1\cdot(p+p_1-k_1) - \frac{1}{2}\sigma^2(p+p_1-k_1)^2}$$

$$\times e^{-iy_2\cdot(p_2-p-k) - \frac{1}{2}\sigma^2(p_2-p-k_2)^2}$$

$$\times e^{-ip^0(x_2^0-x_1^0) - ik_2^0x_2^0 - ik_1^0x_1^0}$$

$$\times e^{ip_2^0x_2^0 + ip_1^0x_1^0}$$

the key phase here is $e^{-ip^0(x_2^0 - x_1^0)}$

V_3

$$e^{-i\sqrt{p^2+m^2}\,L}$$

$$= e^{-iLp\sqrt{\cancel{p^2} + m^2/p^2}}$$

$$= e^{-iLp\left(1+\frac{m^2}{2p^2}\right)}$$

$$= e^{-iLp}\, e^{-iLm^2/2p}$$

$$\Delta\phi = \frac{L\,\Delta m^2}{2p} = \frac{L\,\Delta m^2 c^2}{2\hbar p}.$$