Conditions on $\Pi(q^2)$:

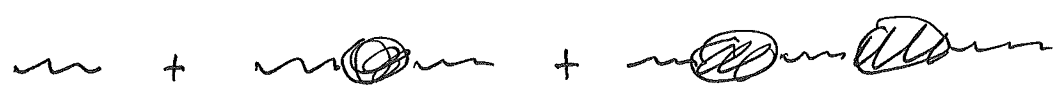
Scalar case. The propagator apart from i and $(2\pi)^4$ is

$$\Delta = \frac{1}{q^2 + m^2 - i\epsilon} =$$ 

The one-particle irreducible graphs sum to Π^*. So

$\Pi^* =$ 

Then the full propagator is Δ'

$\Delta' =$ 

$$= \Delta + \Delta \Pi^* \Delta + \Delta \Pi^* \Delta \Pi^* \Delta$$

$$= \frac{1}{q^2+m^2-i\epsilon} + \frac{1}{q^2+m^2-i\epsilon} \Pi^*(q^2) \frac{1}{q^2+m^2-i\epsilon}$$

$$+ \frac{1}{q^2+m^2-i\epsilon}\left(\Pi^*(q^2)\frac{1}{q^2+m^2-i\epsilon}\right)^2 + \cdots$$

$$= \frac{1}{q^2+m^2-i\epsilon} \frac{1}{\left(1 - \Pi^* \frac{1}{q^2+m^2-i\epsilon}\right)}$$

$$= \frac{1}{q^2+m^2-\Pi^*(q^2)-i\epsilon}$$

We want Δ' to look like $\frac{1}{q^2+m^2-i\epsilon}$ at $q^2=0$.

Say $\quad \pi^*(q^2) = a + bq^2 + c(q^2)^2 + \cdots$

Then Δ' would be

$$\Delta' = \frac{1}{q^2 + m^2 - a - bq^2 - cq^4 + \cdots - i\epsilon}$$

$$= \frac{1}{(1-b)q^2 + m^2 - a - i\epsilon - cq^4 - dq^6 \cdots}$$

which describes a particle of mass squared $m^2 - a$ — the location of the pole — and with residue $1/(1-b)$ instead of unity.

But we know that Δ' describes a particle of mass m, not mass $\sqrt{m^2 - a}$, so $a = 0$. Also, the residue should be unity, so $b = 0$. The other terms, c, d, e, etc., can be non-zero.

For photons $\quad \Delta = \Delta^{\mu\nu} = \dfrac{\eta^{\mu\nu}}{q^2 - i\epsilon}$.

So we are dealing with matrices. We saw that $\pi^*(q^2)$ was of the form

$$\pi^{*\mu\nu}(q^2) = (q^2 \eta^{\mu\nu} - q^\mu q^\nu)\, \pi(q^2). \qquad (11.2.16)$$

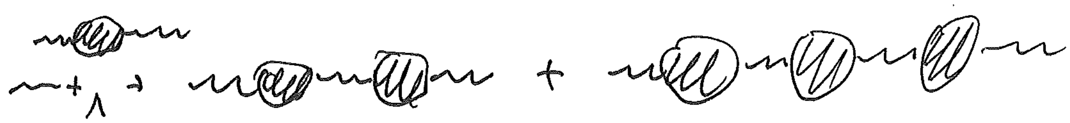
$\Pi^{*\mu}{}_{\nu}$ is a projection operator — apart from the factor π

$$\Pi^{*\mu}{}_{\nu}\,\Pi^{*\nu}{}_{\rho} = (q^2\delta^{\mu}{}_{\nu} - q^{\mu}q_{\nu})\pi(q^2\delta^{\nu}{}_{\rho} - q^{\nu}q_{\rho})\pi$$

$$= \left[(q^2)^2\delta^{\nu}{}_{\rho} - 2q^2 q^{\mu}q_{\rho} + q^{\mu}q_{\rho}q^2\right]\pi^2$$

$$= \Pi^{*\mu}{}_{\rho}\; q^2\pi .$$

The photon case then is

$\Delta'^{\mu}{}_{\nu} =$ 

$$= \Delta^{\mu}{}_{\nu} + \Delta^{\mu}{}_{\rho}\,\Pi^{*\rho}{}_{\sigma}\,\Delta^{\sigma}{}_{\nu} + \Delta^{\mu}{}_{\rho}\,\Pi^{*\rho}{}_{\sigma}\,\Delta^{\sigma}{}_{\varphi}\,\Pi^{*\varphi}{}_{\lambda}\,\Delta^{\lambda}{}_{\nu} + \cdots$$

with $\Delta^{\mu}{}_{\nu} = \dfrac{\delta^{\mu}{}_{\nu}}{q^2 - i\epsilon}$.

Now $\Pi^{*\rho}{}_{\sigma}\,\Delta^{\sigma}{}_{\nu} = \dfrac{\Pi^{*\rho}{}_{\nu}}{q^2 - i\epsilon}$ and so

$$\Pi^{*\rho}{}_{\sigma}\,\Delta^{\sigma}{}_{\varphi}\,\Pi^{*\varphi}{}_{\lambda}\,\Delta^{\lambda}{}_{\nu} = \frac{\Pi^{*\rho}{}_{\varphi}}{q^2 - i\epsilon}\,\frac{\Pi^{*\varphi}{}_{\nu}}{q^2 - i\epsilon} = \frac{\Pi^{*\rho}{}_{\nu}\,q^2\pi}{(q^2 - i\epsilon)^2}$$

$$= \frac{\Pi^{*\rho}{}_{\nu}\,\pi}{q^2 - i\epsilon}$$

So $\Delta'^{\mu}{}_{\nu}$ is

$$\Delta'^{\mu}{}_{\nu} = \frac{\delta^{\nu}{}_{\nu}}{q^2 - i\epsilon} + \frac{\Pi^{*\mu}{}_{\nu}}{(q^2 - i\epsilon)^2} + \frac{\Pi^{*\mu}{}_{\nu} \; \pi q^2}{(q^2 - i\epsilon)^3} + \cdots$$

$$= \frac{\delta^{\mu}{}_{\nu}}{q^2 - i\epsilon} + \frac{\Pi^{*\mu}{}_{\nu}}{(q^2 - i\epsilon)^2}\left[1 + \frac{\pi q^3}{q^2 - i\epsilon} + \left(\frac{\pi q^2}{q^2 - i\epsilon}\right)^2 + \cdots\right]$$

$$= \frac{\delta^{\mu}{}_{\nu}}{q^2 - i\epsilon} + \frac{\Pi^{*\mu}{}_{\nu}}{(q^2 - i\epsilon)^2} \frac{1}{\left(1 - \frac{\pi q^2}{q^2 - i\epsilon}\right)}$$

$$= \frac{\delta^{\mu}{}_{\nu}}{q^2 - i\epsilon} + \frac{\Pi^{*\mu}{}_{\nu}}{(q^2 - i\epsilon)^2 - \pi (q^2 - i\epsilon) q^2}$$

$$= \frac{1}{q^2 - i\epsilon}\left[\delta^{\mu}{}_{\nu} + \frac{\Pi^{*\mu}{}_{\nu}}{q^2 - i\epsilon - \pi q^2}\right]$$

$$= \frac{1}{q^2 - i\epsilon}\left[\delta^{\mu}{}_{\nu} + \frac{(q^2 \delta^{\mu}{}_{\nu} - q^{\mu} q_{\nu})\,\pi}{q^2 - i\epsilon - \pi q^2}\right]$$

$$\Delta'^{\mu}{}_{\nu} = \frac{1}{q^2 - i\epsilon}\left[\delta^{\mu}_{\nu}\left(1 + \frac{q^2\pi}{q^2 - \pi q^2 - i\epsilon}\right) - \frac{q^{\mu} q_{\nu} \pi}{q^2 - \pi q^2 - i\epsilon}\right]$$

$$= \frac{1}{q^2 - i\epsilon}\left[\delta^{\mu}_{\nu}\, \frac{q^2(1-\pi) + q^2\pi - i\epsilon}{q^2(1-\pi) - i\epsilon} - \frac{q^{\mu} q_{\nu}\, \pi}{q^2(1-\pi) - i\epsilon}\right]$$

$$= \frac{1}{q^2 - i\epsilon}\left[\delta^{\mu}{}_{\nu}\, \frac{q^2 - i\epsilon}{q^2 - i\epsilon - \pi q^2} - \frac{q^{\mu} q_{\nu}\, \pi}{q^2(1-\pi) - i\epsilon}\right]$$

$$= \frac{\delta^{\mu}{}_{\nu}}{(1-\pi)q^2 - i\epsilon} - \frac{q^{\mu} q_{\nu}\, \pi}{(q^2 - i\epsilon)(q^2(1-\pi) - i\epsilon)} \qquad (B)$$

The second term doesn't contribute due to current conservation.

To keep the residue of the pole at $q^2 = 0$ at the known value of unity, we need

$$\pi(0) = 0.$$

(B) is equivalent to Srednicki's (62.10) for $\xi = 1$.