$$\mathcal{L} = -\frac{1}{4} F_b^{\mu\nu} F_{b\mu\nu} - \bar{\psi}_b \left[\gamma_\mu (\partial^\mu + i e_b A_b^\mu) + m_b \right] \psi_b$$

$$\psi_b = \sqrt{Z_2}\, \psi \qquad A_b^\mu = \sqrt{Z_3}\, A^\mu$$

$$e_b = e/\sqrt{Z_3} \qquad m_b = m - \delta m$$

$$\mathcal{L} = -\frac{1}{4} Z_3 F^{\mu\nu} F_{\mu\nu} - Z_2 \bar{\psi} \left[\gamma_\mu (\partial^\mu + ie A^\mu) + m - \delta m \right] \psi$$

$$= -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4}(Z_3 - 1) F^{\mu\nu} F_{\mu\nu}$$

$$- \bar{\psi} (\gamma_\mu \partial^\mu + m) - (Z_2 - 1) \bar{\psi} (\gamma_\mu \partial^\mu + m) \psi + Z_2 \delta m \bar{\psi} \psi$$

$$- ie A_\mu \bar{\psi} \gamma^\mu \psi \quad - ie (Z_2 - 1) A_\mu \bar{\psi} \gamma^\mu \psi$$

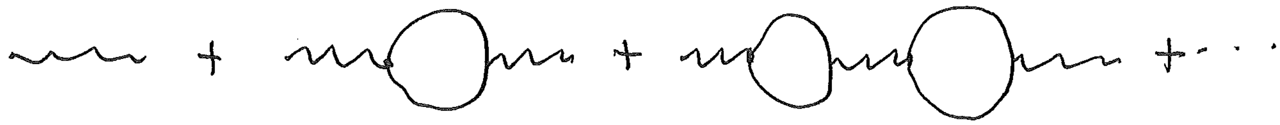
So $\quad \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2$

$$\mathcal{L}_0 = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \bar{\psi} (\not{\partial} + m) \psi$$

$$\mathcal{L}_1 = -ie A_\mu \bar{\psi} \gamma^\mu \psi$$

$$\mathcal{L}_2 = -\frac{1}{4} (Z_3 - 1) F^{\mu\nu} F_{\mu\nu} - (Z_2 - 1) \bar{\psi} (\not{\partial} + m) \psi$$

$$+ Z_2 \delta m \bar{\psi} \psi - ie (Z_2 - 1) A_\mu \bar{\psi} \gamma^\mu \psi$$



$$\Delta + \Delta \Pi^* \Delta + \Delta \Pi^* \Delta \Pi^* \Delta + \cdots$$

$$= \Delta \left(1 + \Pi^* \Delta + (\Pi^* \Delta)^2 + \cdots \right)$$

$$= \Delta \frac{1}{1 - \Pi^* \Delta}$$

We want $\Delta \frac{1}{1 - \Pi^* \Delta} \approx \frac{1}{q^2}$ at $q^2 = 0$

So $\Pi^*(q^2) = 0$.

$$\Gamma\left(2 - \frac{d}{2}\right) \approx \frac{1}{2 - d/2} - \gamma$$

dimensional regularization 't Hooft - Veltman 1976

$$\int d^4 p_e \to \int \Omega_d\, k^{d-1} dk \qquad k = \sqrt{p_e^2}$$

$$\int d^4 p_e\, p^\mu p^\nu \to \int \Omega_d \frac{p^2 \eta^{\mu\nu}}{d} k^{d-1} dk = \int \Omega_d \frac{k^{d+1} \eta^{\mu\nu}}{d} dk$$

$$\int d^4 p_e\, p^\mu p^\nu p^\rho p^\sigma \to \int \Omega_d k^{d-1} dk\, k^4 \frac{[\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}]}{d(d+2)}$$

in which $\Omega_d = \dfrac{2\pi^{d/2}}{\Gamma(d/2)}$

is the area of a unit sphere in d dimensions.

$$\Gamma(z) = \int_0^\infty e^{-x} x^{z-1} dx = (z-1)!$$

$$\Gamma(z+1) = z\,\Gamma(z)$$

$$0! = \Gamma(1) = \int_0^\infty e^{-x} dx = 1$$

$$1! = \Gamma(2) = 1\,\Gamma(1) = 1 \cdot 1 = 1$$

$$2! = 2 = \Gamma(3)$$

$$\Gamma(\tfrac{1}{2}) = \sqrt{\pi} \qquad \Gamma(\tfrac{3}{2}) = \frac{\sqrt{\pi}}{2}$$

$$\Omega_3 = \frac{2\pi^{3/2}}{\Gamma(\frac{3}{2})} = \frac{2\pi^{3/2}}{\sqrt{\pi}/2} = 4\pi, \quad \Omega_4 = \frac{2\pi^2}{\Gamma(2)} = 2\pi^2.$$

$$M^\mu(q) = \int d^4x \, e^{-iqx} \langle \, | J^\mu(x) | \, \rangle$$

$$0 = q_\mu M^\mu = \int d^4x \, i \partial_\mu e^{-iqx} \langle | J^\mu(x) | \rangle$$
$$= -i \int d^4x \, e^{-iqx} \langle | \partial_\mu J^\mu(x) | \rangle = 0.$$

$$M^{\mu\nu}(q,q') = \int d^4x \, d^4x' \, e^{-iqx - iq'x'} \langle | T(J^\mu(x) J^\nu(x')) | \rangle$$

$$q_\mu M^{\mu\nu}(q,q') = \int d^4x \, d^4x' \, i\partial_\mu (e^{-iqx - iq'x'}) \langle | T(J^\mu(x) J^\nu(x')) | \rangle$$
$$= -i \int d^4x \, d^4x' \, e^{-iqx - iq'x'} \langle | \partial_\mu T(J^\mu(x) J^\nu(x')) | \rangle$$

$$T(J^\mu(x) J^\nu(x')) = \theta(x^0 - x'^0) J^\mu(x) J^\nu(x') + \theta(x'^0 - x^0) J^\nu(x') J^\mu(x)$$

So

$$\partial_\mu T(J^\mu(x) J^\nu(x')) = \delta(x^0 - x'^0) J^0(x) J^\nu(x') + \theta(x^0 - x'^0) \partial_\mu J^\mu(x) J^\nu(x')$$
$$- \delta(x'^0 - x^0) J^\nu(x') J^0(x) + \theta(x'^0 - x^0) J^\nu(x') \partial_\mu J^\mu(x)$$

But $\partial_\mu J^\mu(x) = 0$ so

$$\partial_\mu T(J^\mu(x) J^\nu(x')) = \delta(x^0 - x'^0) [J^0(x), J^\nu(x')]$$

$$[J^0(\vec{x}, t), F(\vec{y}, t)] = -q_F F(x,t) \, \delta^3(\vec{x} - \vec{y})$$

current $J^\nu(x')$ is neutral, has $q = 0$ so

$\delta(x^0 - x'^0) [J^\mu(x), J^\nu(x')] = 0$ (except for "Schwinger terms" which don't occur in QED with dim. reg.)