# Physics 523 Homework 8

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## Problem 1

The action density is

$$\mathcal{L} = -\frac{1}{4}F^a_{\alpha\beta}F^{\alpha\beta}_a + \bar{\psi}(i\not{D} - m)\psi$$

with

$$F^a_{\alpha\beta} = \partial_\alpha A^a_\beta - \partial_\beta A^a_\alpha + g f_{abc} A^b_\alpha A^c_\beta$$
$$D_\mu = \partial_\mu - igA^a_\mu t^a$$

so

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial(\partial_\nu A^a_\mu)} &= -\frac{1}{2}(\delta^\nu_\alpha \delta^\mu_\beta - \delta^\nu_\beta \delta^\mu_\alpha)(\partial^\alpha A^\beta_a - \partial^\beta A^\alpha_a + g f^{abc} A^\alpha_b A^\beta_c) \\
&= -\frac{1}{2}(\partial^\nu A^\mu_a - \partial^\mu A^\nu_a + g f^{abc} A^\nu_b A^\mu_c - \partial^\mu A^\nu_a + \partial^\nu A^\mu_a - g f^{abc} A^\mu_b A^\nu_c) \\
&= \partial^\mu A^\nu_a - \partial^\nu A^\mu_a + g f^{abc} A^\mu_b A^\nu_c \\
&= F^{\mu\nu}_a
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial A^a_\mu} &= -\frac{1}{2}F^{\alpha\beta}_d \frac{\partial}{\partial A^a_\mu} g f_{def} A^e_\alpha A^f_\beta + g\bar{\psi}\gamma^\mu t^a \psi \\
&= -\frac{1}{2} g F^{\alpha\beta}_d (f_{dea} A^e_\alpha \delta^\mu_\beta + f_{daf} A^f_\beta \delta^\mu_a) + g\bar{\psi}\gamma^\mu t^a \psi \\
&= -\frac{1}{2} g (f_{dea} F^{\alpha\mu} A^e_\alpha + f_{daf} F^{\mu\beta}) + g\bar{\psi}\gamma^\mu t^a \psi \\
&= g F^{\nu\mu}_c f_{abc} A^b_\nu + g\bar{\psi}\gamma^\mu t^a \psi
\end{aligned}$$

so the Euler-Lagrange equations for the gauge fields are

$$0 = \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu A^a_\nu)} - \frac{\partial \mathcal{L}}{\partial A^a_\nu} = -\partial_\mu F^{\mu\nu}_a - g f_{abc} A^b_\mu F^{\mu\nu}_c - g\bar{\psi}\gamma^\nu t_a \psi.$$

Rearranging:

$$\partial^\mu F^a_{\mu\nu} + g f_{abc} A^{b\mu} F^c_{\mu\nu} = -g\bar{\psi}\gamma_\nu t^a \psi = -g j^a_\nu.$$