Helicity

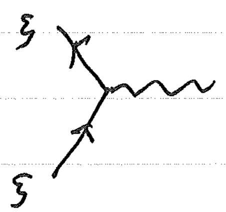
The term $\bar{\Psi}[i\gamma^\mu D_\mu]\Psi = \bar{\Psi}[i\gamma^\mu(\partial_\mu + ieA_\mu)]\Psi$ is the sum of a left-handed piece

$$i\,\xi^\dagger[(\partial_0 + ieA_0)I - (\vec{\nabla} + ie\vec{A})\cdot\vec{\sigma}]\xi$$

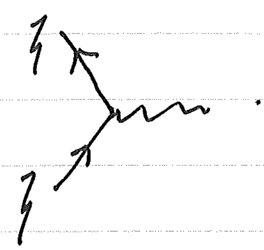
and a right-handed piece

$$i\,\eta^\dagger[(\partial_0 + ieA_0)I + (\vec{\nabla} + ie\vec{A})\cdot\vec{\sigma}]\eta.$$

So the vertex is really two vertices



and



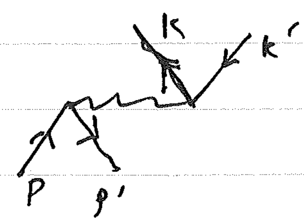
.

The projection operators onto ξ and η are

$$\begin{pmatrix}1&0\\0&0\end{pmatrix} = \frac{1}{2}(I-\gamma^5) \text{ and } \begin{pmatrix}0&0\\0&1\end{pmatrix} = \frac{1}{2}(I+\gamma^5).$$

If we did $e^-e^+ \to \mu^-\mu^+$ with polarized beams of right-handed electrons and left-handed positrons, then we'd replace

$$iM = \frac{ie^2}{q^2}\,\bar{v}(p')\gamma^\mu u(p)\,\bar{u}(k)\gamma_\mu v(k') \text{ for}$$



with

$$\bar{v}(p')\gamma^\mu\left(\frac{1+\gamma^5}{2}\right)u(p)$$

for the electron and positron and use

$$\bar{u}(k)\gamma^\mu\left(\frac{1+\gamma^5}{2}\right)v(k')$$

for the muons if we measured their polarizations as μ^-_R & μ^+_L; P&S show that

$$\sum_{\text{spins}}\left|\bar{v}(p')\gamma^\mu\left(\frac{1+\gamma^5}{2}\right)u(p)\right|^2 = \mathrm{tr}\left[\not{p}'\gamma^\mu\not{p}\gamma^\nu\left(\frac{1+\gamma^5}{2}\right)\right]$$

$$= 2\left(p'^\mu p^\nu + p'^\nu p^\mu - \eta^{\mu\nu}p\cdot p' - i\epsilon^{\alpha\mu\beta\nu}p'_\alpha p_\beta\right)$$

Similarly,

$$\sum_{\text{spins}}\left|\bar{u}(k)\gamma_\mu\left(\frac{1+\gamma^5}{2}\right)v(k')\right|^2 = 2\left(k_\mu k'_\nu + k_\nu k'_\mu - \eta_{\mu\nu}k k' - i\epsilon_{\rho\mu\sigma\nu}k^\rho k'^\sigma\right).$$

Finally, one ~~gets~~ gets (neglecting m_e and m_μ)

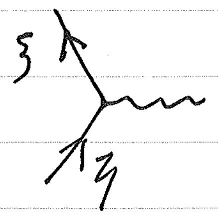
$$|M|^2 = e^4(1+\cos\theta)^2, \text{ and so}$$

$$\frac{d\sigma}{d\Omega}(e^-_R e^+_L \to \mu^-_R \mu^+_L) = \frac{\alpha^2}{4E_{cm}^2}(1+\cos\theta)^2.$$

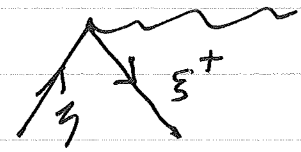
Similarly,

$$\frac{d\sigma}{d\Omega}(e^-_R e^+_L \to \mu^-_L \mu^+_R) = \frac{\alpha^2}{4E_{cm}^2}(1-\cos\theta)^2.$$

Note that the vertices



and



do not exist. So

$$\sigma(e^-_R e^+_R \to \mu^-\mu^+) = 0 .$$

Here ξ and $\sigma_2\eta^*$ are left handed, and $\sigma_2\xi^*$ and η are right handed.