The Phase Transition of Kosterlitz and Thouless

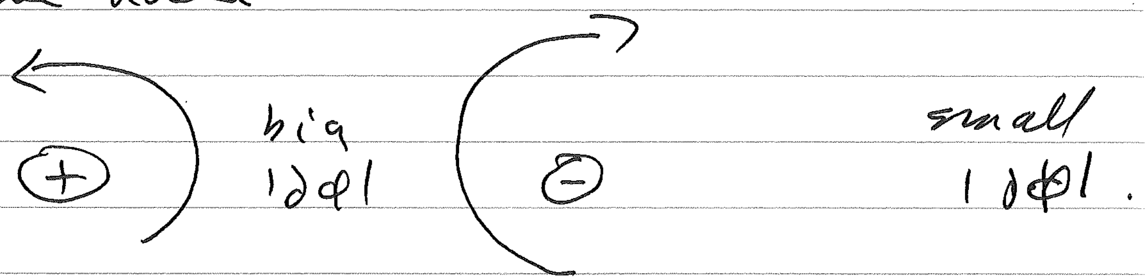
In the (2+1)-dimensional ungauged theory

$$\mathcal{L} = |\partial\phi|^2 - \lambda\,(|\phi|^2 - v^2)^2$$

the vortex $\phi = v e^{i\theta}$ has infinite energy, but not when with an antivortex

$$\phi = v e^{-i\theta}.$$

Now we have



Roughly the energy of such a pair is

$$E \sim v^2 \int d^2x \, \frac{1}{r^2} \sim v^2 \log R/a$$

in which a is the size of the vortex.

Consider the free energy

$$F = E - TS$$

where S is the entropy.

Roughly

$$S = \log L/a$$

where L is the size of the universe.

Let $E = v^2 \log L/a$. Then

$$F = (v^2 - T)\log(L/a).$$

Thus, entropy wins if $T > T_c = v^2$ and the system is a gas of vortices and antivortices. But for $T < T_c$, energy wins, and the vortex-antivortex pairs annihilate or are tightly bound.