Problem (9.23): We start with (9.221)

$$\sqrt{1-a^2} + i\, a\cdot\sigma = \left(\sqrt{1-c^2} + i\, c\cdot\sigma\right)\left(\sqrt{1-b^2} + i\, b\cdot\sigma\right)$$

and find for $2ia_i$ the trace

$$2ia_i = \mathrm{tr}\left[\sigma_i\left(\sqrt{1-c^2} + i\, c\cdot\sigma\right)\left(\sqrt{1-b^2} + i\, b\cdot\sigma\right)\right].$$

$$= \mathrm{tr}\left(\sigma_i \sqrt{1-c^2}\; i b\cdot\sigma\right) + \sqrt{1-b^2}\,\mathrm{tr}\left(\sigma_i\, i c\cdot\sigma\right)$$

$$- \mathrm{tr}\left(\sigma_i\, c_j \sigma_j\, b_k \sigma_k\right)$$

$$= \sqrt{1-c^2}\; 2ib_i + 2ic_i\sqrt{1-b^2}$$

$$- c_j b_k\, \mathrm{tr}\, \sigma_i\left(\delta_{jk} + i\epsilon_{\ell jk}\sigma_\ell\right)$$

$$= \sqrt{1-c^2}\; 2ib_i + 2ic_i\sqrt{1-b^2} - 2i\, \epsilon_{ijk}\, c_j\, b_k$$

So

$$a_i = b_i\sqrt{1-c^2} + c_i\sqrt{1-b^2} + \epsilon_{ijk}\, c_j\, b_k$$

or

$$\vec{a} = \vec{b}\sqrt{1-c^2} + \vec{c}\sqrt{1-b^2} + \vec{c}\times\vec{b}. \qquad (9.222)$$

Problem (9.24): By (9.222), we have

$$a_i = b_i\sqrt{1-c^2} + c_i\sqrt{1-b^2} + \epsilon_{ijk}\, c_j b_k$$

so

$$\frac{\partial a_i}{\partial b_j} = \delta_{ij}\sqrt{1-c^2} + \frac{c_i(-2b_j)}{\sqrt{1-b^2}} + \epsilon_{ilj}\, c_l .$$

At $b=0$, this is

$$\frac{\partial a_i}{\partial b_j} = \delta_{ij}\sqrt{1-c^2} + \epsilon_{ilj}\, c_l .$$

So we must compute the determinant

$$\begin{vmatrix} \sqrt{1-c^2} & -c_3 & c_2 \\ c_3 & \sqrt{1-c^2} & -c_1 \\ -c_2 & c_1 & \sqrt{1-c^2} \end{vmatrix}$$

$$= \sqrt{1-c^2}\left(1-c^2+c_1^2\right) + c_3\left(c_3\sqrt{1-c^2} - c_1c_2\right)$$

$$+ c_2\left(c_3c_1 + c_2\sqrt{1-c^2}\right) = \sqrt{1-c^2} = \sqrt{1-a^2} \text{ at } b=0.$$

So $m(\vec{a}) = 1/\det(\partial a/\partial b)|_{b=0} = \dfrac{1}{\sqrt{1-\vec{a}^2}}$. (9.223)