Brief review of interaction picture,

following Weinberg's Sec. 3.5.

$$S = U(\infty, -\infty)$$

$$U(t, t_0) = e^{iH_0 t} e^{-iH(t-t_0)} e^{-iH_0 t_0}$$

$$i\frac{dU}{dt} = V(t)\, U(t, t_0)$$

$$= e^{iH_0 t}(H - H_0) e^{-iH(t-t_0)} e^{-iH_0 t_0}$$

$$= e^{iH_0 t} V e^{-iH_0 t} e^{iH_0 t} e^{-iH(t-t_0)} e^{-iH_0 t_0}$$

$$= V(t)\, U(t, t_0)$$

So $$U(t, t_0) = T e^{-i\int_{t_0}^{t} V(t')\, dt'}$$

$$U(t, t_0) = 1 - i\int_{t_0}^{t} V(t') U(t', t_0)\, dt'$$

$$= 1 - i\int_{t_0}^{t} V(t_1)\, dt_1 + (-i)^2 \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, V(t_1) V(t_2)$$

$+ \cdots$