By redefining phases of states, we may always absorb a phase and make $k = 1$.

Then absorbing $k^+$

\[ \psi(x) = \phi^+(x) + \phi(x) = \phi^+(x). \]

self

\[ = \phi^+(x) + \phi^+(x)^+ \]

adjoint

\[ [c_N(x), \phi(y)] = 0 \quad \text{for} \quad (x-y)^2 > 0 \]

203

\[ \phi^+(x) = \mathcal{P}(\phi(x)). \]

why?:

After 5210?

We must stick to the convention: we can't use both

\[ \phi = \phi^+ + \phi^- \quad \text{and} \quad \tilde{\phi} = e^{i\phi^+} + e^{-i\phi^-} \]

\[ [\phi(x), \tilde{\phi}(y)] = [e^{i\phi(x)} + e^{-i\phi(x)}] = (e^{-i\phi(x)} - e^{i\phi(x)}) \Delta + (x-y)^2 \Delta \quad \text{for} \quad (x-y)^2 > 0, \]

because they wouldn't commute at $(x-y)^2 > 0$.

One can't use $\phi$ for particles that are charged, because then

\[ [Q, \phi^+(x)] = -\gamma \phi^+(x) \]

\[ [Q, \phi^-(x)] = \gamma \phi^-(x) \]

so

\[ [Q, \phi^+ + \phi^-] = -\gamma (\phi^+ - \phi^-). \]

For charged particles, we need two

\text{spintless bosons of same mass in but charge} \pm \gamma.
\[ [\phi^c(x), \phi^c(x)] = -q \phi^c(x) \]

Then

\[ \phi^c(x) = k \phi^+(x) + \lambda \phi^c(x) = l \phi^c(x) + \lambda \phi^c(x) \]

Now

\[ [\phi^c(x), \phi^c(y)] = 0 \]

while

\[ [\phi^c(x), \phi^c(y)] = [k \phi^+(x) + \lambda \phi^c(x), \phi^c(y)] \]

\[ = (k \Delta_x(y-x) + \lambda \Delta_x(y-x)) \Delta_y(x-y) \quad \text{(general \, x,y)} \]

\[ = (k^2 + \lambda^2) \Delta_x(y-x) \quad \text{for} \, (x-y)^2 > 0. \]

Again we need the field to be bosons.

Needed same mass to get cancellation.

Absorb phase, get

\[ \phi(x) = \phi^+(x) + \phi^-(-x) \]

\[ = \frac{1}{\sqrt{2m^2 - p_0}} \int \frac{d^3p}{2\pi^2} \left[ a(p) e^{ip \cdot x} + a^+(p) e^{-ip \cdot x} \right] \]
We drop back to the neutral case by setting \( a^* = a \).

For a complex or real scalar field \( \phi(x) \) and general \( x, y \):

\[
\begin{align*}
[\phi(x), \phi(y)] &= \Delta(x-y) \\
&= \Delta(x-y) - \Delta(y-x) \\
&= \int \frac{d^3p}{(2\pi)^3 2p_0} \left[ e^{ip\cdot(x-y)} - e^{-ip\cdot(x-y)} \right]
\end{align*}
\]

\[
[\phi(x), \phi(y)] = 0 \quad \forall \quad a \neq a^*.
\]

If \( a = a^* \), then \( \phi(x) = \phi(x) \) and \( [\phi(x, \phi(y)] = [\phi(x), \phi(y)] = \Delta(x-y) \).

A particle carrying no conserved quantum number may or may not be its own antiparticle.

From (4.2):

\[
\begin{align*}
P a^{+}(p) P^{-1} &= \eta a^{+}(-p) \quad \text{(1)} \\
P a^{-}(p) P^{-1} &= \eta^* a^{-}(-p) \quad \text{(2)} \\
P a^{\epsilon+}(p) P^{-1} &= \eta^\epsilon a^{\epsilon+}(-p) \quad \text{(3)}
\end{align*}
\]

Then:

\[
P \phi^{+}(x) P^{-1} = \eta^* \phi^{+}(Px) \quad \text{by (2)}
\]

\[
P \phi^{\epsilon+}(x) P^{-1} = \eta^\epsilon \phi^{\epsilon+}(Px) = \eta \phi^{\epsilon+}(Px) \quad \text{by (3)}
\]

\[
P \phi^{-}(x) P^{-1} = \eta^\epsilon \phi^{\epsilon-}(Px) = \eta \phi^{\epsilon-}(Px)
\]
So \[ P \left[ \phi^+ (x) + \phi^- (x) \right] P^{-1} = \eta^* \phi^+ (p x) + \eta \phi^- (p x) \]
\[ = \phi_p (x) \]

So if we want \( \phi_p (x) = \eta^* \phi (p x) \),
as needed, \( \eta \) must \( \phi (x) \) and \( \phi^+ (x) \) commute, \( \eta \) is \( \frac{1}{2} [\eta (x) + P \eta^+ (x) P^{-1}] \).
Then we need \( \eta \) to satisfy \( \eta^* = \eta \).

\[ P \phi (x) P^{-1} = \eta^* \phi (p x) \]

So the intrinsic parity of a particle and its antiparticle is even, \( |\eta|^2 = 1 \).
If \( \phi = \phi^c \), then \( \eta \) is real: \( \eta = \eta^* = \pm 1 \).

\[ C \]
\[ C \alpha^+ (p) C^{-1} = \bar{\eta} \alpha^+ (p) \]
\[ C \alpha (p) C^{-1} = \bar{\eta} \alpha^c (p) \]
\[ C \alpha^c (p) C^{-1} = \bar{\eta}^c \alpha^+ (p) \]
\[ C \phi (x) C^{-1} = \bar{\eta} \phi^+ (x) = \bar{\eta} \phi (x) \]
\[ C \phi^+ (x) C^{-1} = \bar{\eta}^c \phi^c (x) = \bar{\eta}^c \phi^+ (x) \]
\[ C \phi^{-+} (x) C^{-1} = \bar{\eta} \phi^{-+} (x) = \bar{\eta} \phi^{-+} (x) \]
Let \( C \phi(x) C^{-1} = \xi \phi_c(x) = \xi^* \phi^c(x) + \xi^c \phi^c(-x) \).

To have \( \phi(x) C = C (\phi(x) + \phi^c(x)) C^{-1} = \xi \phi(x) + \xi^c \phi^c(x) \), need \( \xi = [\xi \xi^c] \).

\[
\xi \phi_c(x) = \xi^* \phi^c(x)
\]
\[
\xi \phi(x) = \xi^* \phi^c(x) + \phi^c(-x)
\]

\[\text{we need} \quad \xi = \xi^* \]

Then
\[
C \phi(x) C^{-1} = \xi \phi(x) = \xi^* \phi^c(x)
\]

The intrinsic charge conjugation parity
\( \eta \in \{ +, - \} \) is even.

If \( \phi = \phi^c \), then \( \xi = \xi^* = \pm 1 \) is real.

\( T \)

\[
T a^+(p) T^{-1} = \gamma_a a^+(-p)
\]
\[
T a(p) T^{-1} = \gamma^* a(-p)
\]

By (2.2.8) \( U^+ = U^+ \) for antiunitary ops, too.

\[
T a^+(p) T^{-1} = \gamma^* a^c(-p)
\]
\[ T \phi^+(x) T^{-1} = \gamma^x \int \frac{d^3p}{\sqrt{(2\pi)^3}} a(p) e^{-i \vec{p} \cdot \vec{x}} \]

\[ = \gamma^x \int \frac{d^3p}{\sqrt{(2\pi)^3}} a(p) e^{i \vec{p} \cdot (\vec{x} - \vec{z})} \]

\[ = \gamma^x \int d^3y a(p) e^{i \vec{p} \cdot \vec{y}} \]

\[ \tau_c^+ T \phi^+(x) T^{-1} = \gamma^x \phi^+(\gamma \vec{x}) = \gamma^x \phi^+(\gamma \vec{x}) \]

\[ T \phi^-(x) T^{-1} = \gamma^c \phi^-(\gamma \vec{x}) = \gamma^c \phi^-(\gamma \vec{x}) \]

To have \( T \phi(x) T^{-1} = \gamma^+ \phi(\gamma \vec{x}) = \gamma^+ \phi(-\vec{p} \cdot \vec{x}) \)

we need \( \gamma^c = \gamma^x \). Then

\[ T \phi(x) T^{-1} = \gamma^x \phi(\gamma \vec{x}) = \gamma^x \phi(\gamma \vec{x}) \]

If \( \gamma^c = \gamma \), then \( \gamma = \gamma^x \) is real. \( \gamma = \pm 1 \) if \( \phi^c = \phi \).