$$C\, a^+(p\sigma n)C^{-1} = \xi_n\, a^+(p\sigma n^c) \qquad (4.2.13)$$

$$C\, a(p\sigma n)C^{-1} = \xi_n^*\, a(p\sigma n^c)$$

$$C\, a^{c+}(p\sigma m)C^{-1} = \xi_n^c\, a^{c+}(p\sigma n^c) = \xi_n^c\, a^+(p\sigma n)$$

So

$$C\psi^{(+)}(x)C^{-1} = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}}\, u(p,\sigma) e^{ipx}\, \xi^*\, a^c(p,\sigma)$$

$$C\psi^{(-)c}(x)C^{-1} = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}}\, v(p\sigma) e^{-ipx}\, \xi^c\, a^+(p,\sigma)$$

Now + but keep columns upright

$$\psi^{(-)c\,*}(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}}\, v^*(p,\sigma) e^{ipx}\, a^c(p,\sigma)$$

$$= -\beta C \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}}\, u(p,\sigma) e^{ipx}\, a^c(p,\sigma)$$

$\beta^2 = 1 \qquad C^{-1} = -C \qquad$ so

$$-\xi^* C\beta\psi^{(-)c*}(x) = C\psi^{(+)}(x)C^{-1}$$

And

$$\psi^{(+)*}(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}}\, u^*(p\sigma) e^{-ipx}\, a^+(p,\sigma)$$

$$= -\beta C \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}}\, v(p\sigma) e^{-ipx}\, a^+(p,\sigma)$$

So

$$-\xi^c C\beta\psi^{(+)*} = C\psi^{(-)c}(x)C^{-1}$$

whence

$$C\left(\psi^{(+)}(x) + \psi^{(-)c}(x)\right)C^{-1} = C\beta\left(-\xi^{*}\psi^{(-)c*}(x) - \xi^{c}\psi^{(+)*}(x)\right)$$

So if $\xi^{c} = +\xi^{*}$, then

$$C\,\psi(x)\,C^{-1} = -\xi^{*}\,C\beta\,\psi^{*}(x)$$

Here $*$ means $\dagger$ but keep vectors upright!

If the antiparticle is the same as the particle, then

$$C\,a^{\dagger}(p\sigma n)\,C^{-1} = \xi_n\, a^{\dagger}(p,\sigma,n) \quad \text{and so}$$

$$C\psi(x)\,C^{-1} = \xi^{*}\,\psi(x).$$

In this case

$$C\psi(x)C^{-1} = \xi^{*}\psi(x) = -\xi^{*}C\beta\,\psi^{*}(x) \quad \text{or}$$

$$\psi(x) = -C\beta\,\psi^{*}(x)$$

which is the Majorana condition (5.5.48)