\[ \int \int d\theta \phi (x, y) (\phi_0, e \psi \phi) \]

\[ \int (\xi - \xi') e^{i(t \xi - \xi')} d\theta \phi (x, y) (\phi_0, e \psi \phi) \]

If \( t > 0 \), then we close \( \xi \) contours in \( \text{UV} \)

and get things \( \alpha \to 0 \), \( \epsilon \to \infty \) which \( \to 0 \)

as \( t \to \infty \), as long as \( \psi_\alpha = (\phi_0, e \psi \phi) \)

is analytic near the real \( \xi \), \( \xi' \), axes.

\[ k_0 \mathcal{L}(\xi) - \mathcal{L}(\xi_0) k_0 = -\psi \mathcal{L}(\xi) \]

\[ (k_0 + \omega) \mathcal{L}(\xi) = \mathcal{L}(\xi_0) k_0 \]
\[ \psi_{\pm} = \mathcal{J} \mathcal{L}(\mp \infty) \phi_\alpha \] (\xi, 1.13)

So,

\[ K \psi_{\pm} = K \mathcal{J} \mathcal{L}(\mp \infty) \phi_\alpha \]

\[ = \mathcal{J} \mathcal{L}(\mp \infty) K_0 \phi_\alpha \]

By 13.3.18-19) one has

\[ \tilde{\mathcal{P}} \mathcal{J} \mathcal{L}(\mp \infty) = \mathcal{J} \mathcal{L}(\mp \infty) \tilde{\mathcal{P}}_0 \]

\[ \tilde{\mathcal{J}} \mathcal{L}(\mp \infty) = \mathcal{J} \mathcal{L}(\mp \infty) \tilde{\mathcal{J}}_0 \]

Also, since

\[ H \psi_{\pm} = E_\pm \psi_{\pm} = \mathcal{H} \mathcal{L}(\mp \infty) \phi_\alpha = \mathcal{L}(\mp \infty) \mathcal{H}_0 \phi_\alpha \]

\[ = E_\pm \mathcal{L}(\mp \infty) \phi_\alpha = \mathcal{H} \mathcal{L}(\mp \infty) \mathcal{H}_0 \]

So, \( \psi_{\pm} \) under P.T.'s we get \( \phi_\alpha \)'s.

\[
\begin{pmatrix}
K \\
J \\
P
\end{pmatrix} \mathcal{L} = \mathcal{L} \begin{pmatrix}
K \\
J \\
P
\end{pmatrix}_0
\]

\[
\begin{pmatrix}
K \\
J \\
P
\end{pmatrix} \psi_{\pm} = \mathcal{L} \begin{pmatrix}
K \\
J \\
P
\end{pmatrix}_0 \phi_\alpha.
\]
Internal Symmetries

They act as unitary linear transformations on species indices

\[ U(T) \psi_{p, q, m; p_0, m_0} = \sum_{\tilde{m}, \tilde{m}_0} \mathcal{D}^{(1)}_{\tilde{m}, \tilde{m}_0} (T) \mathcal{D}^{(2)}_{\tilde{m}, \tilde{m}_0} (T) \]

\[ \times \frac{1}{4} p_1 \tilde{m}_1 p_2 \tilde{m}_2 \]

The \( U \)'s obey the group rules of multiplication

\[ U(T) U(T) = U(TT) \]

and the \( \mathcal{D} \)'s form representations of the group

\[ \mathcal{D}(TT) \mathcal{D}(T) = \mathcal{D}(T) \]

\[ (\psi_m^+ , \psi_m^-) = (U(T) \psi_m^+ , U(T) \psi_m^-) = \sum_{\tilde{m}, \tilde{m}_0} \mathcal{D}^{(1)}_{m, \tilde{m}} (T) \mathcal{D}^{(2)}_{m, \tilde{m}_0} (T) (\psi_{\tilde{m}}, \psi_{\tilde{m}_0}) \]

\[ (U(T) \psi_{\alpha}^-, U(T) \psi_{\alpha}^+) = \sum_{\tilde{N}, \tilde{N}_0} \mathcal{D}_{\tilde{N}, \tilde{N}_0}^*(T) \mathcal{D}_{\tilde{N}, \tilde{N}_0}^*(T) = \sum_{\tilde{N}, \tilde{N}_0} \mathcal{D}_{\tilde{N}, \tilde{N}_0}^*(T) \mathcal{D}_{\tilde{N}, \tilde{N}_0}^*(T) \]

and so unitary

\[ \sum_{\tilde{N}_1, \tilde{N}_2, \tilde{N}_1', \tilde{N}_2'} \mathcal{D}_{\tilde{N}_1, \tilde{N}_1'} (T) \mathcal{D}_{\tilde{N}_2, \tilde{N}_2'} (T) \]

\[ \times \sum_{p_1, \sigma_1, \tilde{N}_1, p_2, \sigma_2, \tilde{N}_2} p_0, \tilde{N}_1, \tilde{N}_2 \]

\[ = \sum_{p_1, \sigma_1, \tilde{n}_1, p_2, \sigma_2, \tilde{n}_2} p_0, \tilde{n}_1, \tilde{n}_2 \]

\[ = (\psi_{\alpha}^-, \psi_{\alpha}^+) \]

(3.3.33)
provided the same U(1) induces these transformations on the $Q_0^+$'s and the $Q_0^-$'s.

Such a symmetry is a symmetry of the theory if on the free states $\exists U_0(T)$

$$U_0(T)\phi_{m_1,m_2,...} = \sum \Theta(T) \Theta(T) \phi_{N_1,N_2,...}$$

and

$$U_0^{-1}(T) H_0 U_0(T) = H_0$$

$$U_0^{-1}(T) V U_0(T) = V.$$ 

For from $U_0^{-1}(T) M U_0(T) = M$ and so by $(3.1.13)$

$$\psi^\pm_\alpha = \mathcal{J}_{\alpha}(T \to \infty) \phi_\alpha = e^{\pm i H_MT_B + i H_0 T_B}$$

we have

$$U_0(T)\psi^\pm_\alpha = \mathcal{J}_{\alpha}(T \to \infty) U_0(T) \phi_\alpha$$

So we can use $U_0(T) = U(T)$ to get $(3.3.29).$
The U(1) case: single parameter $\theta$

$$T(\theta) T(\Theta) = T(\Theta + \theta)$$

We saw in (2.2.24-26) that

$$U[\text{LT}(\theta)] = \exp(i\theta Q)$$

with $Q^+ = Q$.

We use as a basis the eigenstates of $Q$:

$$Q \Psi_m = \delta_m^0 \Psi_m;$$

then

$$\left(\Psi_m, U[\text{LT}(\theta)] \Psi_m\right) = (\Psi_m, e^{i\theta Q} \Psi_m) = (\Psi_m, \delta^0_m \Psi_m) = \delta^0_m \delta^{0m}$$

$$= (\Psi_m, \delta [\text{LT}(\theta)] m m' \Psi_{m'}) = \delta [\text{LT}(\theta)] m m$$

So by (3.3.33) with $\delta [\text{LT}(\theta)] m m = \delta m m \exp(i\theta \delta m n)$,

$$S_{\beta\alpha} = (\Psi_\beta, \Psi_\alpha^+) = (U[\text{LT}(\theta)] \Psi_\beta, U[\text{LT}(\theta)] \Psi_\alpha^+) = (e^{i\theta Q} \Psi_\beta, e^{i\theta Q} \Psi_\alpha^+)$$

$$= e^{i\theta (Q^m m - Q^m m')} (\Psi_\beta, \Psi_\alpha^+) = e^{i\theta (Q^m m - Q^m m')} S_{\beta\alpha}$$

So

$$\sum_i \delta m_i = \sum_i \delta m_i'$$

$$q_{\alpha} = q_{\alpha'}$$ or $S_{\alpha \alpha'} = 0$.

Conservation of charge.
E.g., electric charge or baryon number = 
\[ \# p's + \# n's + \# \text{hyperons} - \# \bar{p}'s - \# \bar{n}'s - \# \text{hyperon}'s, \]
and lepton number = \[ N_e + N_\mu + N_\tau + N_\nu - N_{\bar{e}} - N_{\bar{\mu}} - N_{\bar{\tau}} - N_{\bar{\nu}}. \]

But baryon and lepton number are probably only approximately conserved, see Vol. II.

Strange mesons are conserved only by the strong and electromagnetic interactions, not by the weak ones: \[ S = n_s^5 - m_s. \]

Rochester + Butler 1947 saw \( K^+ \)'s and \( \Lambda^0 \)'s in cosmic rays;

These decayed slowly,

\[ K^+, K^0 \quad S = +1 \]
\[ K^- \bar{K}^0 \quad S = -1 \]
\[ \Sigma^+, \Xi^0, \Xi^- \quad \Lambda^0 \]

\[ p \rightarrow \pi^+ \pi^- \pi^- \quad S = 0 \]
\[ \pi^+ \rightarrow K^+ \Lambda^0 \quad \text{fast in strong interactions} \quad 10^{-23} \]

But both \( \Lambda^0 \rightarrow p \pi^- \) and \( K^+ \rightarrow \pi^+ \pi^- \) are slow:

\[ \tau(\Lambda^0) \approx 2.63 \times 10^{-8} \]
\[ \tau(K^+) \approx 1.237 \times 10^{-8} \]

while \[ \tau(p) \approx 10^{-23} \]

\[ \Gamma(p) \approx 151 \text{ MeV} \]

\[ p \rightarrow \pi^+ \pi^- \pi^- \quad m_p \approx 770 \text{ MeV} \]

\[ T = \frac{t}{2 \tau} = \frac{6.6 \times 10^{-22} \text{ MeV s}}{151 \text{ MeV}} = \frac{10^{-22}}{2.6} \approx 5 \times 10^{-24} \]
A non-abelian symmetry is one based on a non-abelian group—one whose generators do not all commute.

Isotopic spin 1937 when strong py forces were found similar to strong cp force.

\[ G = SU(2) \text{ covering group of } SO(3) \]

\[ \ell_i, \ell_j, j = 1, 2, 3 \]

Exact SU(2) requires that particles form degenerate multiplets with \( T \) integer/2 and \( 2T+1 \) components of \( T_3 \). E.g., \( \mu, \nu \) have \( T = \frac{1}{2} \)

\[ \pm \frac{1}{2} \quad \pi^+, \pi^0, \pi^- \text{ have } T = 1, \epsilon_3 = 1, 0, -1. \]

\( \Lambda^0 \) has \( T = t_3 = 0. \)

\[ Q = t_3 + \frac{B+S}{2} \]

Gell-Mann, Ne'eman 1960

\( T \) and \( Y = B+S \) are generators of \( SU(3) \).

Now seen just as a consequence:

\[ \frac{m_u}{\Lambda}, \frac{m_d}{\Lambda}, \frac{m_s}{\Lambda} < 1. \]
One may use $SU(2)_I$ like $SU(2)_W$.

(3.3.33) implies for $A + B \to C + D$ that approximately

$$\begin{align*}
S_{\Delta^D, \Delta^A, \Delta^B} &= \sum_{T_C} \frac{C_{T_C, T_D}}{C_{T, T_3}} (T_{3,3}; t_3 t_3) \\
&\times C_{T_A, T_B} (T_{3,3}; t_3 t_3) S_T
\end{align*}$$

where $C_{j, j_0; \alpha, \alpha_0}$ is the Clebsch-Gordan coefficient for forming spin $j_0$ from $j, j_0, j_0$. This $S_T$ is a "reduced" S-matrix depending on $T$ and on kinematics etc., but not on $t_3^C = ABC^\dagger d_3$.

$SU(2)_I$ is not respected by the electroweak interactions. Indeed $j_\mu_1$, $m_1$ have different charges and masses.

$$SU(3)_C \times SU(2)_L \times U(1)_{em}$$

See C-T VII, p. 1048, eq. (2)

$$S = \sum_{j_\mu_1} P(j, j_\mu_1) S_{\mu_1} P(j_\mu_1) = \sum_{j_\mu_1} \frac{1}{r_{j_\mu_1}^3} \frac{1}{r_{j_\mu_1}^3} S_{\mu_1}$$

$$\langle j_\mu_1 m_1 \mu_1 | s_1 j_1 m_1 \mu_1 \rangle = \sum_{j_\mu_1} \langle j_\mu_1 m_1 \mu_1 | j_1 m_1 \mu_1 \rangle \langle j_1 m_1 \mu_1 | s_1 j_1 m_1 \mu_1 \rangle$$
Parity

Approximately, $\tilde{x} \rightarrow -x$. Were it valid, there would be a unitary $P$ acting on $\xi^+$ and $\xi^-$

$$
P \frac{\gamma}{\gamma', m^2} \xi^+ \xi^- = \eta_1 \eta_2 \eta_3 \eta_4 \xi^+ \xi^- \frac{\gamma}{\gamma', m^2} \eta_1 \eta_2 \eta_3 \eta_4 \xi^+ \xi^-
$$

for massive particles. Then

$$
(\xi^+, \xi^-) = (P \xi^+, P \xi^-)
$$

$$
P \text{ will anti } J \text{ of } P_0
$$

$$
P_0 \Gamma_0 P_0^{-1} = \Gamma_0
$$

$$
P_0 \Gamma P_0^{-1} = \Gamma
$$

$$
P_0 \phi = \eta_1 \phi \phi
$$

The $\eta$'s are the intrinsic parities.

$$
S_{\phi \phi} = \eta_1 \eta_2 \eta_3 \phi \phi
$$