So

$$-\frac{d}{dt}\int d\alpha \frac{V^{N_\alpha}}{(2\pi)^{3N_\alpha}} P_\alpha \ln P_\alpha$$

$$\geq \int d\alpha\, d\beta \frac{V^{N_\beta}}{(2\pi)^{3N_\beta}} \frac{d\Gamma(\beta\to\alpha)}{d\alpha}\left[P_\beta - P_\alpha\right]$$

$$\geq \int d\alpha\, d\beta\, P_\alpha \left[\frac{V^{N_\alpha}}{(2\pi)^{3N_\alpha}} \frac{d\Gamma(\alpha\to\beta)}{d\beta} - \frac{V^{N_\beta}}{(2\pi)^{3N_\beta}} \frac{d\Gamma(\beta\to\alpha)}{d\alpha}\right]$$

$= 0$ by (3.6.18)

ie

$$-\frac{d}{dt}\int \frac{d\alpha\, V^{N_\alpha}}{(2\pi)^{3N_\alpha}} P_\alpha \ln P_\alpha \geq 0$$

Boltzmann H-theorem.

When $P_\alpha = P_\alpha(E, P, Q, B, L)$ is a function of the various conserved quantities, then

$\frac{d\Gamma(\beta\to\alpha)}{d\alpha} = 0$ unless $P_\alpha = P_\beta$

eg. if $P_\alpha = Q_\alpha$, then $\alpha\to\beta$ is possible only if $Q_\alpha = Q_\beta$

In this case we may set $P_\alpha = P_\beta$ in (3.6.19):

$$\frac{V^{N_\alpha}}{(2\pi)^{3N_\alpha}} \frac{dP_\alpha}{dt} = P_\alpha \int d\beta \left[\frac{V^{N_\beta}}{(2\pi)^{3N_\beta}} \frac{d\Gamma(\beta \to \alpha)}{d\alpha} - \frac{V^{N_\alpha}}{(2\pi)^{3N_\alpha}} \frac{d\Gamma(\alpha \to \beta)}{d\beta} \right]$$

$$= 0 \text{ by } (3.6.18)$$

ie then $\frac{dP_\alpha}{dt} = 0$ and the growth of entropy stops when $P_\alpha = P_\alpha(Q_\alpha, \bar{E}_\alpha)$ etc.