Scattering of Identical Particles

We recall that in our formula

$$e^{ikz} + f_k(\theta,\varphi)\frac{e^{ikr}}{r} \qquad (1)$$

the variables z and r referred to the relative position

$$r = |\vec{r}_1 - \vec{r}_2|$$

$$z = z_1 - z_2.$$

If we interchange the two particles, then

$$\theta \to \pi - \theta \qquad \text{and}$$
$$\varphi \to \varphi + \pi.$$

Consider the scattering of two spinless particles. The space wave-function must be symmetric, so instead of (1) we need something like

$$e^{ikz} + e^{-ikz} + [f_k(\theta,\varphi) + f_k(\pi-\theta,\varphi+\pi)]\frac{e^{ikr}}{r}.$$

So

$$\frac{d\sigma}{d\Omega} = |f_k(\theta,\varphi) + f_k(\pi-\theta,\varphi+\pi)|^2.$$

That is

$$\frac{d\sigma}{d\Omega} = |f_k(\theta,\varphi)|^2 + |f_k(\pi-\theta,\varphi+\pi)|^2$$
$$+ 2\,\mathrm{Re}\left[f_k(\theta,\varphi)\, f_k^*(\pi-\theta,\varphi+\pi)\right].$$

If f_k is φ-independent, then this cross-section is enhanced at $\theta = \pi/2$:

$$\frac{d\sigma}{d\Omega} = 2\,|f_k(\tfrac{\pi}{2})|^2 + 2\,\mathrm{Re}\,|f_k(\tfrac{\pi}{2})|^2 = 4\,|f_k(\tfrac{\pi}{2})|^2.$$

Consider now the scattering of two identical particles of spin 1/2. Suppose both beams are spin polarized with spin up. Then the spin state is $|{+}{+}\rangle$ which is symmetric. The space state then must be antisymmetric, and so the differential cross-section is

$$\frac{d\sigma}{d\Omega} = |f_k(\theta,\varphi) - f_k(\pi-\theta,\varphi+\pi)|^2$$
$$= |f_k(\theta,\varphi)|^2 + |f_k(\pi-\theta,\varphi+\pi)|^2$$
$$- 2\,\mathrm{Re}\left[f_k(\theta,\varphi)\, f_k^*(\pi-\theta,\varphi+\pi)\right].$$

If $\frac{\partial f_k(\theta,\phi)}{\partial \phi} = 0$, then at $\theta = \frac{\pi}{2}$

$$\frac{d\sigma}{d\Omega} = 2\,|f_k(\tfrac{\pi}{2})|^2 - 2\,|f_k(\tfrac{\pi}{2})|^2 = 0 \quad !$$

There is no scattering at $\theta = \frac{\pi}{2}$!.

Unpolarized beams of spin $\frac{1}{2}$ particles contain 3 triplet states ($|++\rangle$, $(|+-\rangle + |-+\rangle)/2$, $|--\rangle$) for every singlet state ($(|+-\rangle - |-+\rangle)/2$). So now if $\partial f/\partial\phi = 0$

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\,|f_k(\theta) + f_k(\pi-\theta)|^2 + \frac{3}{4}\,|f_k(\theta) - f_k(\pi-\theta)|^2$$

$$= |f_k(\theta)|^2 + |f_k(\pi-\theta)|^2 - Re[f_k(\theta) f_k^*(\pi-\theta)].$$

At $\theta = \pi/2$, this is

$$\frac{d\sigma}{d\Omega} = 2\,|f_k(\tfrac{\pi}{2})|^2 - |f_k(\tfrac{\pi}{2})|^2$$

$$= |f_k(\tfrac{\pi}{2})|^2$$

and the suppression is less than in the polarized triplet case.