Fifth homework assignment tentatively due on Wed 22 April.

1. Consider the scattering of a (spinless) particle off a square-well potential as in the class notes. Assume that $k_0 r_0$ is very close to an odd multiple of $\pi/2$. Set

$$k_0 r_0 = (2n+1)\,\frac{\pi}{2} + \epsilon \tag{1}$$

with $|\epsilon| \ll 1$. Show that if $\epsilon > 0$, then there is a bound state with energy

$$E = -\frac{(\hbar k'')^2}{2\mu} \tag{2}$$

with $k'' \approx \epsilon k_0$.

2. Now in the previous problem change the sign of ϵ, so that $\epsilon < 0$. Show that there is a bump in the s-wave total x-section (a scattering resonance) at energy

$$E = \frac{(\hbar k)^2}{2\mu} \tag{3}$$

with

$$k^2 \approx -\frac{2k_0\epsilon}{r_0}. \tag{4}$$

In effect, as the depth of the potential well is reduced, bound states become scattering resonances.

3. Use a computer (or your fingers) to graph the total s-wave x-section $\sigma_0(k)$ as a function of k for $k_0 r_0 = (2n+1)\pi/2 \pm \epsilon$. Do this for a few values of $n \leq 4$.

4. Use equations (27) and (33) of the class notes "Light and Atoms in SI Units" to derive Eq.(36) of those notes.