

Fourth homework assignment tentatively due on Wed 8 April.

1. Apply the variational method to find the ground state of the generic hamiltonian

$$H = H_0 + V \quad (1)$$

using trial states of the form

$$|a_1, \dots, a_n\rangle = \sum_{j=1}^{\infty} a_j |E_j\rangle \quad (2)$$

in which the state  $|E_j\rangle$  is an eigenstate of  $H_0$

$$H_0 |E_j\rangle = E_j |E_j\rangle \quad (3)$$

with energy  $E_j$ . Interpret your result.

2. Suppose two identical spin-one particles are both in s-states in some potential. If they also have the same value of the principal quantum number  $n$ , *i.e.*, they are in the same space state, what are the possible values of the total angular momentum  $j$ ?
3. Consider two identical spin-one-half particles in an box of side  $L$  with a potential  $V_0$  that is zero inside the box and infinite outside it. (a) If the particles do not interact, what are the energies and wave-functions (space and spin)? (b) Suppose now the potential is

$$V(\mathbf{r}_1, \mathbf{r}_2) = V_0(\mathbf{r}_1, \mathbf{r}_2) + a^2 \delta^{(3)}(\mathbf{r}_1 - \mathbf{r}_2) \quad (4)$$

in which  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are the positions of the two particles. Show that the triplet states (*i.e.*, those of total spin  $\hbar$ ) are unaffected by the change in the potential.