Fourth homework assignment tentatively due on Wed 8 April.

1. Apply the variational method to find the ground state of the generic hamiltonian

\[ H = H_0 + V \]  

using trial states of the form

\[ |a_1, \ldots a_n\rangle = \sum_{j=1}^{\infty} a_j |E_j\rangle \]  

in which the state \(|E_j\rangle\) is an eigenstate of \(H_0\)

\[ H_0 |E_j\rangle = E_j |E_j\rangle \]

with energy \(E_j\). Interpret your result.

2. Suppose two identical spin-one particles are both in s-states in some potential. If they also have the same value of the principal quantum number \(n\), i.e., they are in the same space state, what are the possible values of the total angular momentum \(j\)?

3. Consider two identical spin-one-half particles in an box of side \(L\) with a potential \(V_0\) that is zero inside the box and infinite outside it. (a) If the particles do not interact, what are the energies and wave-functions (space and spin)? (b) Suppose now the potential is

\[ V(r_1, r_2) = V_0(r_1, r_2) + a^2 \delta^{(3)}(r_1 - r_2) \]

in which \(r_1\) and \(r_2\) are the positions of the two particles. Show that the triplet states (i.e., those of total spin \(\hat{S}\)) are unaffected by the change in the potential.