Second homework assignment tentatively due on Wed 18 Feb.

1. Consider an electron with orbital angular momentum $\ell = 1$ (and of course spin 1/2) in a state in which the total angular momentum $J = 3/2$ and $M = 1/2$. (a) What is the probability that the spin of the electron is up? (b) Same question, but if $J = 1/2$?

2. The Δ resonance occurs in pion-nucleon scattering at about 1232 MeV. The pion has isopin 1, so that $|\pi^+\rangle = |1,1\rangle$, $|\pi^0\rangle = |1,0\rangle$, and $|\pi^-\rangle = |1,-1\rangle$. The proton-neutron system has isospin 1/2 with $|p\rangle = |1/2,1/2\rangle$ and $|n\rangle = |1/2,1/2\rangle$. This giant resonance has $J = 3/2$ and isospin $I = 3/2$. The $I_z = 1/2$ state of this resonance can decay into both $|\pi^0,p\rangle$ and $|\pi^+,n\rangle$. What is the ratio of these two decay modes?

3. (a) Compute the derivatives of the unit vectors $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$, and $\hat{\boldsymbol{\phi}}$ with respect to the variables r, θ, and ϕ. Your formulas should express these derivatives in terms of the basis vectors $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$, and $\hat{\boldsymbol{\phi}}$. (b) Using the formulas of (a), derive a formula for the laplacian $\boldsymbol{\nabla}\cdot\boldsymbol{\nabla}$.

4. Consider the two-body hamiltonian

$$H = \frac{\boldsymbol{p}_1^2}{2m_1} + \frac{\boldsymbol{p}_2^2}{2m_2} + V(\boldsymbol{r}_1 - \boldsymbol{r}_2). \tag{1}$$

Define the total momentum as

$$\boldsymbol{P} = \boldsymbol{p}_1 + \boldsymbol{p}_2 \tag{2}$$

the relative momentum as

$$\boldsymbol{p} = \frac{m_2\,\boldsymbol{p}_1 - m_1\,\boldsymbol{p}_2}{m_1 + m_2}. \tag{3}$$

and the separation as

$$\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2. \tag{4}$$

Now show that H can be written as

$$H = \frac{\boldsymbol{P}^2}{2M} + \frac{\boldsymbol{p}^2}{2\mu} + V(\boldsymbol{r}) \tag{5}$$

in which $M = m_1 + m_2$ is the total mass, and

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \tag{6}$$

is the reduced mass.

5. Using the same notation as in the last problem and the definition

$$\boldsymbol{R} = \frac{m_1\,\boldsymbol{r}_1 + m_2\,\boldsymbol{r}_2}{m_1 + m_2} \tag{7}$$

of the center of mass, show that the total orbital angular momentum of the two bodies

$$\boldsymbol{L} = \boldsymbol{L}_1 + \boldsymbol{L}_2 = \boldsymbol{r}_1 \times \boldsymbol{p}_1 + \boldsymbol{r}_2 \times \boldsymbol{p}_2 \tag{8}$$

can be written as

$$\boldsymbol{L} = \boldsymbol{R} \times \boldsymbol{P} + \boldsymbol{r} \times \boldsymbol{p}. \tag{9}$$