First homework assignment tentatively due on Wed 4 Feb.

1. For an eigenstate $|E\rangle$ of the hamiltonian

$$H = \frac{\boldsymbol{p}^2}{2m} + g^2 \left(x^4 + y^4 + z^4 \right) \tag{1}$$

   what is the relation between the mean value of the kinetic energy and that of the potential energy $V = g^2 \left(x^4 + y^4 + z^4 \right)$?

2. Same problem but with

$$V = g^2 \left(x^n + y^n + z^n \right). \tag{2}$$

3. Find the eigenvalues and eigenvectors of

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{3}$$

   Consider an electron in the state

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha|+\rangle + \beta|-\rangle. \tag{4}$$

   If we measure $S_2 = (\hbar/2)\sigma_2$, what is the probability of the result $\hbar/2$?

4. The spin-dependent part of the hamiltonian H of an electron and a positron in a uniform magnetic field B in the z-direction is

$$H = A\boldsymbol{S}_e \cdot \boldsymbol{S}_p + \frac{eB}{mc} \left(S_{ze} - S_{zp} \right). \tag{5}$$

   Consider the state $|+,-\rangle = |+_e, -_p\rangle$ in which the electron is "spin up" and the proton "spin down." (**a**): Is this state an eigenstate of H in the limit $A \to 0$ with $eB/mc \neq 0$? If yes, what is its energy eigenvalue? If no, what is the mean value of H in this state? (**b**): Same problem in the opposite limit, $eB/mc \to 0$ with $A \neq 0$.

5. If $\boldsymbol{S}$ is the spin operator of a massive, spin-one particle, what are the matrix elements of the operators

$$S_z(S_z - \hbar)(S_z + \hbar) \quad \text{and} \quad S_y(S_y - \hbar)(S_y + \hbar) \tag{6}$$

   between its various states?