

2 x 2 notation matrices

$$e^{-i\theta \cdot \sigma} = 1 - i\frac{\theta \cdot \sigma}{2} - \frac{(\theta \cdot \sigma)^2}{2 \cdot 2^2} + i\frac{(\theta \cdot \sigma)^3}{3! \cdot 2^3} + \dots$$

$$(\theta \cdot \sigma)^2 = \theta_i \theta_j \sigma_i \sigma_j = \theta_i \theta_j (\delta_{ij} + i\epsilon_{ijk} \sigma_k)$$

$$= \theta^2 \quad \text{so with } \theta = \sqrt{\vec{\theta} \cdot \vec{\theta}}$$

$$e^{-i\frac{\theta \cdot \sigma}{2}} = 1 - i\frac{\theta \cdot \sigma}{2} - \frac{1}{2} \left(\frac{\theta}{2}\right)^2 + i\frac{\left(\frac{\theta}{2}\right)^2 \theta \cdot \sigma}{3! \cdot 2}$$

$$= \cos \frac{\theta}{2} - i\hat{\theta} \cdot \sigma \left(\frac{\theta}{2} - \frac{1}{3!} \left(\frac{\theta}{2}\right)^3\right)$$

$$= \cos \frac{\theta}{2} - i\hat{\theta} \cdot \sigma \sin \frac{\theta}{2}$$

3 x 3 notation matrices

$$e^{-i\theta \cdot \mathbf{J}} = \cos \theta - i\hat{\theta} \cdot \vec{\mathbf{J}} \sin \theta + (1 - \cos \theta) \hat{\theta} \hat{\theta}^T$$

$$(\mathbf{J}_k)_{ij} = i\epsilon_{ijk}$$

$$\left(e^{-i\theta \cdot \mathbf{J}} \right)_{ij} = \delta_{ij} \cos \theta - \sin \theta \epsilon_{ijk} \hat{\theta}_k + (1 - \cos \theta) \theta_i \theta_j$$