Special problem 8.3

The interaction of an external (classical) current density \( \tilde{J}(x,t) \) with the electromagnetic field \( \tilde{A}(x,t) \) is

\[
V = -\frac{1}{c} \int \mathbf{d}^3x \tilde{J}(x,t) \cdot \mathbf{A}(x,t).
\]

So the vacuum state \( |0\rangle \) will evolve into

\[
|j,t\rangle = \mathcal{T} \left\{ e^{\int_0^t \frac{1}{c} \int \mathbf{d}^3x \mathbf{A}(x,t') \cdot \tilde{J}(x,t') dt'} \right\} |0\rangle
\]

in the interaction picture at time \( t > 0 \).

Recall the identity

\[
\mathbf{A} \cdot \mathbf{B} + \frac{i}{2} \{ \mathbf{A}, \mathbf{B} \} = \mathbf{C} \cdot \mathbf{C} = \mathbf{C}
\]

which holds when \( \{ \mathbf{A}, \mathbf{B} \} \) commutes with both \( \mathbf{A} \) and \( \mathbf{B} \). When one uses it to write

\[
|j,t\rangle = \mathcal{T} \left\{ e^{\frac{i}{c} \int \mathbf{d}^3x \mathbf{A}(x,t') \cdot \tilde{J}(x,t') dt'} \right\} |0\rangle
\]

without the time ordering, one must deal with commutators like

\[
Y = \left[ \int \mathbf{d}^3x \mathbf{A}(x,t') \cdot \tilde{J}(x,t') \right] \left[ \int \mathbf{d}^3y \mathbf{A}(y,t') \cdot \tilde{J}(y,t') \right]
\]
Is $Y$ an operator or merely a complex number (multiplied by the identity operator)?

If $Y$ is an operator, is it hermitian, anti-hermitian, or neither?

If $Y$ is merely a complex number (called a c-number), is it real, imaginary, or neither?

You may assume that $\hat{A}$ is hermitian, and that $\hat{j}$ is real.