Special problem 8.3

The interaction of an external (classical) current density $\vec{j}(x,t)$ with the electromagnetic field $\vec{A}(x,t)$ is

$$V = -\frac{1}{c}\int d^3x \, j(x,t)\cdot A(x,t). \tag{1}$$

So the vacuum state $|0\rangle$ will evolve into

$$|j,t\rangle_I = T\left\{ e^{+\frac{i}{\hbar c}\int_0^t d^3x \, A(x,t')\cdot j(x,t')\,dt'} \right\}|0\rangle \tag{2}$$

in the interaction picture at time $t>0$.

Recall the identity

$$e^A e^B = e^{A+B+\frac{1}{2}[A,B]} \tag{3}$$

which holds when $[A,B]$ commutes with both A and B. When one uses it to write $|j,t\rangle_I$ in the form

$$|j,t\rangle_I = e^{\frac{i}{\hbar c}\int_0^t d^3x dt' \, \vec{A}(x,t')\cdot\vec{j}(x,t') + X} \tag{4}$$

without the time ordering, one must deal with commutators like

$$Y = \left[\int d^3x \, \vec{A}(\vec{x},t')\cdot\vec{j}(\vec{x},t'), \int d^3y \, \vec{A}(\vec{y},t'')\cdot\vec{j}(\vec{y},t'')\right]. \tag{5}$$

Is Y an operator or merely a complex number (multiplied by the identity operator)?

If Y is an operator, is it hermitian, anti-hermitian, or neither?

If Y is merely a complex number (called a c-number), is it real, imaginary, or neither?

You may assume that $\vec{A}$ is hermitian, and that $\vec{j}$ is real.