Polarized Light in Calcite

In vacuum, a photon's wave-function is

$$\langle x, t | p E \rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{i(p\cdot x - Et)/\hbar} .$$

Let's use the abbreviations

$$\vec{k} = \vec{p}/\hbar \quad \& \quad \omega = E/\hbar ,$$

Then

$$\langle x, t | \vec{k} \rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{i(k\cdot x - \omega t)} .$$

Since the photon is massless, $E = pc$. And also, by Einstein's formula, $E = h\nu = \hbar\omega$. So in vacuum

$$|\vec{k}| = \frac{p}{\hbar} = \frac{E}{\hbar c} = \frac{\hbar\omega}{c} = \frac{\omega}{c} .$$

But in matter, $\vec{k}$ also depends upon the index of refraction $n(\vec{k})$

$$|\vec{k}| = n(\vec{k})\,\omega/c .$$

A birefringent crystal, like calcite, has an optic axis. Photons polarized $\perp$ to the optic axis are ordinary rays. Those polarized $\parallel$ to the optic axis are extraordinary rays.

Red light in calcite has $n_o \simeq 1.1 n_e$, so

$$|k_o| = n_o \frac{\omega}{c} \quad \text{and}$$

$$|k_e| = n_e \frac{\omega}{c}$$

and $|k_o| \simeq 1.1 |k_e|$.

The phase velocity is had from

$$k \cdot dx - \omega\, dt = 0$$

so that $v_p = \dfrac{dx}{dt} = \dfrac{\omega}{k} = \dfrac{\omega}{n\omega/c} = \dfrac{c}{n(\hat{k})}$,

So the ordinary rays are 10% slower than the extraordinary rays.

Say the photon comes into the calcite as $|in\rangle$,

$$|in\rangle = |e\rangle\langle e|in\rangle + |o\rangle\langle o|in\rangle.$$

If the plate is of thickness ℓ, then

$$\langle \ell, t|out\rangle = \langle \ell, t|e\rangle\langle e|in\rangle + \langle \ell t|o\rangle\langle o|in\rangle$$

$$= \left(e^{ik_e \ell} \langle e|in\rangle + e^{ik_o \ell} \langle o|in\rangle \right) \frac{e^{-i\omega t}}{\sqrt{2\pi\hbar}}.$$

The relative phase is

$$(k_o - k_e)\ell$$

so

$$|out\rangle \approx |e\rangle + e^{i(k_o - k_e)\ell} |o\rangle$$

A $\lambda/4$-plate has a thickness such that

$$(k_o - k_e)\ell = \frac{\pi}{2}.$$

In the demo, I had linearly polarized light $|in\rangle$. I oriented the $\lambda/4$-plate at $\pm 45°$ to the plane of polarization of the laser light, so that the in state was

$$|in\rangle = |e\rangle\langle e|in\rangle \pm |o\rangle\langle o|in\rangle$$

in which $\langle e|in\rangle$ and $\langle o|in\rangle$ were about $1/\sqrt{2}$. The out state was

$$|out\rangle = \frac{|e\rangle \pm i|o\rangle}{\sqrt{2}}$$

apart from a phase. This light is right or left circularly polarized.

If instead, I turn the $\frac{\lambda}{4}$-plate so that

$$|in\rangle = |e\rangle$$

or

$$|in\rangle = |o\rangle,$$

then

$$|out\rangle = |e\rangle$$

or

$$|out\rangle = |o\rangle.$$