Polarized Light in Calcite

In vacuum, a photon's wave function is

\[ \langle x, t | \psi \rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{i(p \cdot x - E t)/\hbar} \]

Let's use the abbreviations

\[ \vec{k} = \frac{p}{\hbar} \quad \text{and} \quad \omega = \frac{E}{\hbar} \]

Then

\[ \langle x, t | \vec{k} \rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{i(k \cdot x - \omega t)} \]

Since the photon is massless, \( E = pc \).

And also, by Einstein's formula, \( E = h\nu = \omega c \).

So in vacuum

\[ |k|^2 = \frac{p^2}{\hbar^2} = \frac{E^2}{\hbar^2 c^2} = \frac{\hbar\nu}{c} = \frac{\omega}{c} \]

But in matter, \( |k|^2 \) also depends upon the index of refraction \( n(k) \)

\[ |\vec{k}|^2 = n(k) \omega c \]

A birefringent crystal, like calcite, has an optic axis. Photons polarized \( \perp \) to the optic axis are ordinary rays. Those polarized \( \parallel \) to the optic axis are extraordinary rays.
Red light in calcite has \( n_0 \approx 1.1 \) Me, so

\[ 1 \, \text{kol} = m_0 \omega^2 \]

and

\[ 1 \, \text{kel} = m_0 \frac{\omega}{c} \]

and \( 1 \, \text{kol} \approx 1.1 \, 1 \, \text{kel} \).

The phase velocity is had from

\[ k \cdot dx - \omega dt = 0 \]

so that

\[ U_p = \frac{dx}{dt} = \frac{\omega}{k} = \frac{\omega}{n_0 w/c} = \frac{\omega}{n(k^2)} \]

So the ordinary rays are 10\% slower than the extraordinary rays.

Say the photon comes into the calcite as \( |1\text{in}\rangle \).

\[ |1\text{in}\rangle = 1e x |1\text{in}\rangle + 10 x 0 |1\text{in}\rangle \]

If the plate is of thickness \( l \), then

\[ \langle 2, \text{out} \rangle = \langle 2, \text{in} | e x |1\text{in}\rangle + \langle 0 | e x |1\text{in}\rangle \]

\[ = \left( e^{-i k_0 l} \frac{i k_0}{\sqrt{2 \pi k}} + e^{-i k_0 l} \frac{i k_0}{\sqrt{2 \pi k}} \right) \frac{e^{-i \omega^2}}{\sqrt{2 \pi k}} \].
The relative phase is

\[(k_0 - k_e)l\]

so

\[i(k_0 - k_e)l\]

\[10\psi > \approx 1e > + e 10 >\]

A \(\lambda/4\)-plate has a thickness such that

\[(k_0 - k_e)l = \frac{\pi}{2}\]

In the demo, I had linearly polarized light \(1m >\) I oriented the \(\lambda/4\)-plate at \(\pm 45^\circ\) to the plane of polarization of the laser light, so that the incident state was

\[1in > = 1e \times 1e 1in > \pm 10 \times 0 1in >\]

in which \(\langle e1m >\) and \(\langle 01m >\) were about \(\sqrt{2}\).

The output state was

\[10\psi > = \frac{1e > + \langle 10 >}{\sqrt{2}}\]

apart from a phase. This light is right or left circularly polarized.
Instead, I turn the \( \frac{1}{4} \) plate so that

\[ 1w > = 1e > \]

or

\[ 1w > = 107 > \]

then

\[ 100w > = 1e > \]

or

\[ 100w > = 107 > \]